STUDY OF RADIATIVE AND ELECTROMAGNETIC DISCONTINUITIES

THESIS PRESENTED

By

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M.Sc. (Physics & Maths)

for

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(MATHEMATICS)

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CERTIFICATE

This is to certify that the work embodied in the thesis entitled **"STUDY OF RADIATIVE AND ELECTROMAGNETIC DISCONTINUITIES"** being submitted by **BRIJENDRA KUMAR RATHORE** M.Sc. (Maths & Physics) for the award of the degree of Doctor of Philosophy of the Bundelkhand University, Jhansi (U.P.), has been carried out under my supervision and guidance, that the work embodied has not been submitted elsewhere for the award of any other degree and is up to the mark both in its academic contents and the quality of presentation.

It is further certified that **BRIJENDRA KUMAR RATHORE** has attended the department for more than two hundred days.

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B. K. Rathore

Date 23.12.2000 **BRIJENDRA KUMAR RATHORE**

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PREFACE

The present thesis an out come of researches carried out by me in the field of "STUDY OF RADIATIVE & ELECTROMAGNETIC DISCONTINUTIES" under the supervision of Dr. S. P. Singh Bhadauria, M.Sc., Ph.D. Reader & Head, Deptt. of Maths, Pt. J. N. College, Banda (U.P.) affiliated to Bundelkhand University, Jhansi, is being submitted for the award of Ph.D. degree in Mathematics. This thesis has been devided into seven chapters, each chapter has been subdevided into a number of articles.

The first chapter is introductory. It gives in brief, an idea about shock waves, Lagrangian Method, Eulerian Method, equation of state, equation of continuity, shock wave their existance, spherical and cylindrical shock wave, equation of motion and jump condition in magnetohydrodynamics, radiation phenomenon, radiant pressure, radiation flux, similarity principle and self similar gas motion and concept of self gravitation.

The second chapter is devoted to unify study of PROPAGATION OF PLANE RADIATIVE SHOCK WAVE IN UNSTABLE HOMOGENOUS MEDIUM taking disturbance headed by an radioactive shock surface of a rotating thunder in the earth medium and definded study is made for plane shock wave.

The Third chapter deals with ANALYTICAL SOLUTION OF SELF GRAVITATING GAS BEHIND A CYLINDRICAL SHOCK WAVE IN MAGNETOGASDYNAMICS. The motion of gas is assumed under azimuthal magnetic field, with gas in self gravitating. Energy of wave increases with time. An analytical study is made for cylindrical shock wave with azimuthal magnetic field.

In fourth chapter, an attempt is made for the ANALYTICAL SOLUTION OF SPHERICAL SHOCK WAVE IN A ROTATING GAS WITH AXIAL COMPONENT OF MAGNETIC FIELD, taking density as constant ahead the shock front, By using C.C.W. method, analytical solution are have been obtained for shock velocity and shock strength for weak and strong magnetic field. For strong shock, also we have considered two cases i.e. when the magnetic field is strong and when weak i.e. nonmagnetic case.

Fifth chapter deals with ANALYSIS OF MAGNETO HYDRODYNAMICS SPHERICALLY SHOCK WAVE IN A MEDIUM WITH VARIABLE DENSITY, energy is suddenly released with spherical symmetry in a conducting gas, under the effect of azimuthal magnetic field where density varies with time. Numerical integration of differential equation is carried out in DEC 1090, Computer System at IIT, Kanpur by well known R.K.G's program

In sixth chapter, an attempt is made for the study of PROPAGATION OF PLANE SHOCK WAVE IN MAGNETO GAS DYNAMICS under isothermal condition where temperature gradient is zero and radiation effect have been taken out, Numerical integration of differential equation is carried out on DEC 1090, Computer at IIT Kanpur, by R.K.G's program.

In seventh chapter, an attempt is made for the analytical solution of RADIATIVE EFFECT OF POINT EXPLOSION IN STELLAR BODIES. Since the temperature of material is very high the radiation effect can not be ignore. The wave is produce by sudden point explosion. The effect of transverse magnetic field be taken out.

CONTENTS

CHAPTER - 1

INTRODUCTION

The study of 'Fluid Dynamics" is directed to the behavior of a fluid in motion. The liquid and gas states are referred to generally as fluids. Some of the notable examples of application are : flow of water through pipes; motion of an aircrafts or a missiles in the atmosphere. The study also yields the methods and devices for the measurement of various parameters, e.g. the pressure and velocity in a fluid at rest or in motion.

The first notable works on fluids appeared in the seventeenth century. In 1687, Newton in his book 'PRINCIPIA' deals with the Influx of a fluid, the resistance of a fluid and the resistance of projected bodies and the wave motion. Daniel Bernoulli in 1738 determined the relation between pressure and velocity and formalized this in his theorem. In 1743 his son John Bernoulli applied the momentum principle to infinitesimal elements. Euler realized the significance of the Bernoulli's work and used it as the basis, setting down the fundamental equations of motion and equation of countinuity for an ideal inviscid fluid in 1755, An alternate form of these equations was given by Lagrange in 1781 and 1789. Equations including viscosity were derived by Navier in 1822 and stokes in 1845.

In 1858 Helmholtz published a paper on Vortex motion and in 1868 another on free stream line potential flow.

Near the end of the nineteenth century. Fluid flow itself began to be extensively observed and investigated. The modern trend in fluid dynamics is to investigate the flow of electrically conducting fluid at very high temperature.

At a microscopic level, the three states of a given substance are different because of the difference in the intermolecular distance. In many

cases at normal conditions the molecular distance of a fluid are less than the minutest of any physical dimensions of practical interest. As a result, we interested in the statistical average properties and the behavior of large numbers of the molecules, and not in that of individual molecules. That is, macroscopic and not microscopic, properties are of interest. In fluid dynamics as individual molecules are not being considered, the fluid can be regarded as a continuous medium and so the physical quantities such as mass, momentum etc. of the fluid contained in a very small volume are regarded as being spread uniformly throughout that volume.

In dealing with the gases at very low pressure, as in the upper atmosphere, or at very high temperatures such as in a plasma. The continuum concept of fluid dynamics must be vilotated and the study that has to be based on the behavior of individual molecules (i.e. on the macroscopic approach)

In our studies we frequently refer to a 'small element of fluid' which is always supposed so large that it still contains a very large number of molecules, as fluid in a continuous medium. So when we take of an infinitesimal element of volume we always mean that which are physically infinite small. Such an element is called a fluid particle.

In continuum dynamics we assume that the macroscopic fluid properties, for example mean density, mean pressure, very continuously with (a) the size of element of fluid considered (b) the position in the fluid, and (c) the time in (a), the variation becomes negligible as the element is physically very small. Thus, fluid properties density, pressure and velocity are expressed as continuous functions of position and time only. On this basis, it is possible to establish equation governing the motion of a fluid, which are independent of the nature of the particle structure. So gases and liquids may be treated together.

There are two distinct methods of specifying the flow field.

(i) LAGRANGIAN METHOD

In this method, the flow variables (velocity, pressure and density) of a selected fluid element or particle are described. If $\bar{r_0}$ is the position of the center of mass of the fluid element at time t_0 then the basic flow quantity in the Lagrangian description is the velocity $\overline{\nabla}(r_0, t)$. This method is also referred as 'individual time rate of change.

(ii) EULERIAN METHOD

In this method, the flow quantities are described at all points of space occupied by the fluid at all times i.e. flow quantities are defined as the function of position in space (r) and time (t). The basic flow quantity is the vector velocity $\overline{\nabla}$ (r, t). This method corresponds to 'local time rate of change'.

Consider any scalar function f (x,y,z, t) associated with some property of the fluid (it could be density, velocity etc.) Suppose fluid particle has the position p (x,y,z) or p (r) at time t. Keeping this point fixed the change in f is during the interval of time δt is

$$f(x, y, z, t + \delta t) - f(x, y, z, t)$$

Hence the 'local time rate of change' is given by

$$\frac{\partial f}{\partial t} = \lim_{\delta t \to 0} \frac{f(x, y, z, t + \delta t) - f(x, y, z, t)}{\delta t}$$

As the point P is fixed the local time differential operator $\delta f / \delta t$ is not carried along by the moving fluid.

Now let at time $t + \delta t$ the fluid particle, which was at the position (x, y, z) originally, is in the position ($x + u\delta t$), $y + v\delta t$, $z + w\delta t$) where u, v and w be the

velocity components at the position at time t. The corresponding change of f is given by.

$$f(x + u\delta t\ ,\ y + v\delta t,\ z + w\delta t,\ t + \delta t) - f\ (x,\ y,\ z, t)$$

or $f(\bar{r}+\delta\bar{r},\ t + \delta t) - f(\bar{r},\ t)$

and rate of change is,

$$\frac{df}{dt} = \lim_{\delta t \to 0} \frac{f\ (r + \delta r,\ t + \delta t) - f\ (r, t)}{\delta t}$$

This given the individual time rate of change as the point p is moving t gives the rate of change which is carried along by the moving fluid.

Since,

$$f = f\ (x,\ y,\ z,\ t)$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial t} dt$$

So,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

If $\bar{V} = [u,v,w]$ be the velocity of the fluid particle at p and

$$\frac{dx}{dt} = U, \qquad \frac{dy}{dt} = V, \qquad \frac{dz}{dt} = W$$

Then

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}u + \frac{\partial f}{\partial y}v + \frac{\partial f}{\partial z}w$$

$$= \frac{\partial f}{\partial t} + \left(i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}\right)(ui + vj + wk)$$

or

$$\frac{df}{dt} = \frac{\partial f}{\partial t}(\bar{V}.\nabla)f$$

Where $V = ui + vj + wk$

The first term on the right hand side represents the local rate of change of f and the second term the convective rate of change.

If $\bar{V}$ is introduced for f in the above equation then the total derivative of velocity with respect to time is,

$$a = \frac{d\bar{v}}{dt} = \frac{\partial\bar{v}}{\partial t} + (\bar{V}.\nabla)\bar{V}$$

In cartesion rectangular coordinates

$$(\bar{V}.\nabla) = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

It is this term in the Euler relation for acceleration which is responsible for the non linearities in the equations of motion of fluid dynamics.

EQUATION OF STATE

The electro dynamical state relation is a simple relation between the current density J, fields and fluid motion, or the ohm's law is,

$$\overline{J} = \sigma \{ \overline{E} + \mu\overline{V} + \overline{H} \} + q\,\overline{v}$$

Where σ is electrical conductivity and q V is the current depending on the motion of the net charge q.

The state of a compressible fluid is defined by the pressure P. the entsopy S, the internal energy E, the absolute temperature T. the mass density ρ and the specific volume V.

An equation of state is formed by expressing only one of the thermodynamic variables in terms of the other two quantities mentioned above.

for example,

$$P = p(\rho, T)$$

or $$E = E(P, \rho)$$

for a perfect fluid (no intermolecular interactions), having negligible viscosity the equation is

$$P = R\rho T$$

Where, R is the universal gas constant. Assuming the process to be adiabatic and isentropic we have,

$$\varepsilon = C_V T$$

$\varepsilon = P / R \rho C_v$ as $P = \rho R T$

Also

$$C_P - C_V = R$$

or

$$\frac{C_P}{C_V} - 1 = \frac{R}{C_V}$$

or

$$\gamma - 1 = \frac{R}{C_V}$$

$$\therefore \quad C_V = \frac{R}{\gamma - 1}$$

$$\varepsilon = \frac{P}{\rho(\gamma - 1)}$$

This is the caloric equation of state of the medium. is the internal energy per unit mass.

When a fluid element changes its state isentropically, the state equation is

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$

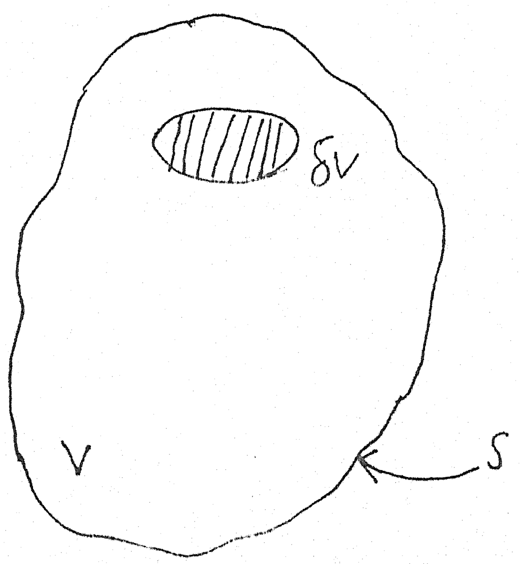
EQUATION OF CONTINUITY

This equation simply expresses the law of conservation of mass.

The quantity of fluid entering a certain volume in space must be balanced by that quantity leaving i.e. matter is neither created nor destroyed.

We now formulate this principle mathematically.

Let V be any arbitrary volume fixed in space, bounded by a surface S, and containing a fluid of density ρ. The volume element δv is small so that ρ can be regarded as constant through it



The mass of the fluid with in the volume V is $\int_v \rho dv$ the rate of generation of the fluid with in the volume is

$$\frac{\partial}{\partial t}\int_v \rho dv = \int_v \frac{\partial \rho}{\partial t} dv$$

for $\rho \dfrac{\partial}{\partial t}(dv) = \rho d\left(\dfrac{\partial v}{\partial t}\right) = \rho_0\, 0 = 0$ as volume is constant with respect to time.

If the volume V occupied by a moving fluid, the fluid enters V through parts of its boundary surface S and leaves through another part.

Let $\hat{n}$ be a unit outward normal vector drawn on the surface element ds. The normal velocity in n.v. The total outward flux is

$$\int_s \rho\, (\hat{\eta} \cdot \bar{v})\, ds = \int_s \hat{\eta} \cdot (\rho\, \bar{v})\, ds$$

using Gauss's theorem

$$= \int_v \nabla \cdot (\rho\, \bar{v})\, dv$$

The sum of the net outward convection of mass plus the rate of generation of the fluid with in the volume must be zero.

$$\int_v \nabla \cdot \ (\rho \bar{V}) \ \ dv + \int_v \frac{\partial \rho}{\partial t} dv = 0$$

or

$$\int_v \left[\ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \bar{v}) \right] dv = 0$$

Since this is true for arbitrary elementary volumes

$$\frac{\partial \rho}{\partial t} \ + \ \nabla \cdot (\rho\, \bar{v}) = 0$$

or

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho\, v_i) = 0 \qquad\qquad i = 1,2,3$$

The Maxwell's electromagnetic equations for a conducting medium are,

$$\text{Curl } \overline{E} = \frac{\partial \overline{B}}{\partial t} \qquad \text{where } \overline{B} = \mu\, \overline{H}$$

$$\text{Curl } \overline{H} = 4\,\pi\,\overline{J} + \frac{\partial \overline{D}}{\partial t}$$

$$\text{div } \overline{B} = 0$$

and $\quad \text{div } \overline{D} = 4\,\pi\, q$

Differentiating the fourth Maxwell's equation

$$\text{div}\left(\frac{\partial \overline{D}}{\partial t}\right) = 4\,\pi\,\frac{\partial q}{\partial t}$$

using the second Maxwell's equation it becomes,

$$\text{div}\,(\text{Curl } \overline{H} - 4\,\pi\,\overline{j}\,) = 4\,\pi\,\frac{\partial q}{\partial t}$$

Third Maxwell's equation is

$$\text{div}\,\overline{H} = 0$$

we have

$$-4\pi\,\text{div}\,\overline{J} = 4\pi\,\frac{\partial q}{\partial t}$$

or

$$\frac{\partial q}{\partial t} + \text{div}\,\overline{J} = 0$$

If the charged particle moves with velocity v

$$\overline{J} = q\overline{v} \qquad \text{then,}$$

$$\frac{\partial q}{\partial t} + \text{div}\,(q\,\overline{V}) = 0$$

This is the equation of continuity for the electric charge moving under the effect of magnetic field which is similar to the equation.

SHOCK WAVE, THEIR EXISTENCE

If a small disturbance (i.e. with an infinitesimal amplitude and small velocity), is created within a non viscous isentropic compressible fluid it will propagate throughout the fluid as a wave motion and with the velocity of sound relative to the fluid without suffering any distortion.

In this case we get the wave equations, which shows that both the density and velocity variations follow the same wave patterns, derived from the linearizing of the equations of fluid motion. The solution ρ (x,t) and u (x,t)

both are the single valued function. The equations of fluid motion reduced to linear form as velocity is small, so that the term $(\bar{v}\,.\,\text{grad})\,\bar{v}$ in Euler's equation of motion may be neglected.

If the assumptions of infinitesimal amplitudes and gradients are removed, then the wave velocity will not be a constant and a simple wave velocity will distort as it propagates.

In this case as velocity is not small, we have to solve the complete non linear equations of fluid motion. The method of characteristics can be applied to solve such hyperbolic type equations.

A simple analytic solution of these equations of fluid motion may be find such that the density is a function of velocity only. After some manipulation we get equations in u and ρ. General solutions of these equations are,

$$u = f_1\,[\,x - (\,u \pm a)\,t\,]$$

$$\rho = f_2\,[\,x - (\,u \pm a)\,t\,]$$

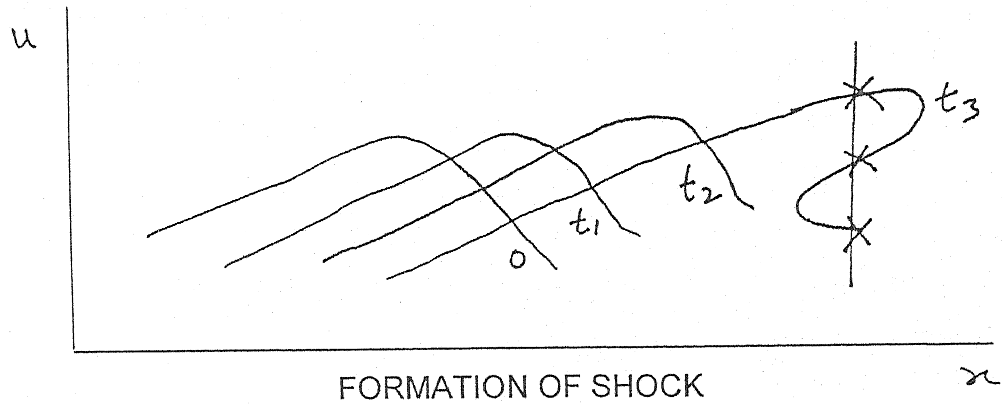
where f_1 and f_2 are arbitrary functions.

we consider the solution,

$$u = f_1\,[\,x - (u + a)\,t\,]$$

This equation shows that the disturbance is propagated at an instantaneous velocity u + a, instantaneous because this velocity is a function of time. If the increment in velocity u is much smaller then speed of sound, then the solution is the case of sound wave i.e. the linear case and the curve $u = r_1\,(x - at)$ does not change its shape as the disturbance propagates. but if u is not small the shape of the wave is distorted as it propagates.

The resulting distortions in velocity distribution are shown in figure.



FORMATION OF SHOCK

At a time t_1, the points of high velocity move faster to the right than those of lower velocity. Thus the creast has moved faster to the right than the through and so the profile is disturbed. The compressive part of the wave, where the propagation velocity is a decreasing function of X, distorts to give a triple valued solution for u (x, t), which is physically impossible for longitudinal wave.

Actually, the difficulty is over come by the formation of a shock wave. In such cases, a large variation in pressure and density occurs in a very narrow region in which the flow variables change rapidly and the fluid no longer undergoes isentropic changes. The thickness, of this shock wave region is very small and so we may considered the shock wave as a surface of discontinuity for many practical problems of inviscid fluids.

The propagation of shock is faster than sound when observed from one side of the discontinuity and less than that of sound when observed from the other side. Hence the velocity of the shock will be supersonic viewed from ahead and it will be subsonic viewed from behind.

The formation of these shock waves causes a great noise equal to that of an explosion, usually termed as supersonic bang so the effect of shock must be taken into account in the design of aeroplanes, pipe flow, supersonic flight of projectiles and so on.

For the flow of an inviscid and non conducting gas the laws of conservation of mass, momentum and energy are originally formulated in the differential equation form as it is assumed that the flow variables defining the flow are continuous functions. Flows are also possible, however, for which discontinuities in the distribution of these flow variables occur. So the conservation laws can also be applied to such discontinuous flow and hence across a shock.

The condition for the existence of shock waves which may be called the jump conditions, relate the velocity pressure, density and temperature in front of the shock to those behind of them. The jump conditions are the simple consequence of the laws of conservation of mass, momentum and energy across the surface of discontinuity and the equation of state of the medium through which the shock is moving.

SPHERICAL AND CYLINDRICAL SHOCK WAVE

Consider the propagation of a shock wave, through a perfect gas, of great intensity (i.e. very strong) resulting from a strong explosion i.e. from the instantaneous release of a large quantity of energy.

When the energy is suddenly released, in an infinitely concentrated form and distribution of density, pressure etc. depends only on the distance from some point then this is the case of spherical shock wave.

When the energy is suddenly released along a line and distribution of all quantities in homogenous in same direction and has complete axial symmetry about that direction, then this is the case of cylindrical shock.

Since we consider the symmetrical flow (centrally or axially) so there are only two independent variables, namely r and t. So the one – dimensional fundamental equations governing the adiabatic flow are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + j\frac{u}{r}\right) = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{1}{\rho}\frac{\partial \rho}{\partial r} = 0$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\frac{\rho}{\rho^{\gamma}} = 0$$

or

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0$$

Where j = 0,1 or 2 stands for plane, cylindrical or spherical case, respectively, and r is the radial distance from the centre in spherical case, the radial distance from the line of explosion, in the cylindrical case and x coordinate in plane case; u is radial velocity; ρ the density.

Similarity principle may be used to reduce these equations to ordinary differential equations.

EQUATION OF MOTION AND JUMP CONDITION IN MAGNETO HYDRODYNAMICS

In an electrically conducting fluids in the presence of magnetic field discontinuity i.e. shock wave in flow variable can exists.

The study of magneto hydrodynamic shock waves was begun in 1950 with the paper of F. de Hoffmann and Taylor.

When electric current induced in the fluid, then their flow in the magnetic field produces mechanical forces which modify the motion, magneto gas dynamic owes its peculiar interest and difficulty to this interaction between the field and the motion. Thus the equations of magneto gas dynamic are the ordinary and electromagnetic equations. We only take into account the interaction between the motion and the magnetic field, we have ignored the Maxwell's displacement currents. Through out the thesis we take $\mu = 1$ because magnetic permeability (μ) differs only slightly from unity which is unimportant. We also assume that the dissipative mechanism such as viscosity, thermal conductivity and electrical resistant are absent.

Since the problems dealt in this thesis relate to magneto gas dynamic or magneto – radiative shocks. We refer in this article the relevant flow and fluid equation.

$$P = \rho R T :$$

$$\frac{D\rho}{Dt} + \rho\frac{\partial U}{\partial r} + J\frac{\rho u}{r} = 0$$

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{\partial h^2}{\partial r} = 0$$

$$\frac{Dh}{Dt} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{\partial hu}{r} = 0$$

$$\frac{\partial}{\partial t}(P\rho^{-r}) + u\frac{\partial}{\partial r}(P\rho^{-r}) = 0$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$$

and u, p, ρ, h, r and t are the velocity, pressure, density, magnetic field transverse to the flow, radial distance and time respectively, T is the ratio of specific heat at constant volume and constant pressure, J=0, 1 and 2 corresponding to plane, cylindrical and spherical respectively and υ = 0 for plane and υ = 1 corresponds to cylindrical and spherical both.

In the presence of magnetic field, the relation connecting the flow and field quantities on the two sides of the shock surface are as follows, where the velocity in front the shock wave is zero

$$h_2 (u - u_2) = h_1 u ;$$

$$\rho_2 (u - u_2) = \rho_1 u ;$$

$$P_2 + \frac{1}{2} h_2^2 + \rho_2 (u - u_2)^2 = p_1 + \frac{1}{2} h_1^2 + \rho_1 u^2;$$

$$\frac{1}{2}(u - u_2)^2 + \frac{r\,p_2}{(r-1)\rho_2} + \frac{h_2^2}{\rho_2} = \frac{1}{2}u^2 + \frac{r\,p_1}{(r-1)\rho_1} + \frac{h_1^2}{\rho_1}$$

where suffix 1 and 2 correspond to the volume of the quantities just ahead and just behind the shock surface and u is the shock velocity.

RADIATION PHENOMENON

At very high temperature, the gases become ionized and radiation can be considered as a continuous emmision of energy is the form of electromagnetic wave which propagated in the medium with the speed of light. This energy is called radiant of thermal energy radiation where as according to quantum theory the radiant energy emitted or absorbed is not continuous, permitting all possible values, as demanded by the wave theory, but in a discrete quantified form, as integral multiples of an elementary quantum of energy, photon or light quanta. The amount of energy in each quanta being given by the product $h\upsilon$. where h is planck's constant and υ is the frequency of the radiation.

Thus the quantum theory proposes the particles characteristic of radiation, while classical theory the wave characteristics, both being required to understand the complex behavior of radiation. A complete study of such a high temperature flow of a gas should consist of the study of gas dynamic field the electromagnetic field and thermal radiation flux simultaneously. The analysis of simultaneously effects and magnetic field forms the subject matter of radiation magneto hydrodynamics. This theory of thermal radiation can be applied to understand the processes which take place in stellar media, to explain the observed luminosity of stars and nuclear explosions and also to high temperature flow.

Radiative transfer and radiative heat exchange have an influence on both the star and the motion of the fluid. this influence is caused by the fact that fluid loses or gains energy by emitting or observing heat. On the flow field of the gas there are three radiation effects expressed in terms of radiation pressure, radiation energy and radiation flux

RADIANT PRESSURE

By the theory of electro dynamic the pressure of a radiation field is equal to one third of radiant energy is

$$P_R = \frac{1}{3} E_r = \frac{1}{3} a_R T^4 :$$

where T is temperature and a_R is Stefan – Boltzmann constant. This is the only component of the radiation which differs from zero.

RADIATION ENERGY

The radiant energy density Er per unit mass of the fluid is given by

$$Er = \frac{a_R T^4}{\rho}$$

where ρ is density of fluid.

RADIATION FLUX

The net amount of radiant energy passing through the surface per unit area per unit time is called the radiant flux through the surface is given by

$$F_R = \frac{C_R}{4}, \frac{a_R T^4}{\rho}$$

or $\quad F_R = D_R \text{ grad } E_R$

where $D_R = \frac{CL_R}{3}$ is the Rosseland diffusion coefficient of radiation and $L_R = \frac{1}{K\rho}$ the Rosseland mean free path of radiation.

Magnetic pressure P_h and the magnetic energy E_h are given by

$$P_h = \frac{h^2}{2p} ;$$

$$E_h = \frac{h^2}{2p} ;$$

where h is the magnetic field.

The fundamental equation of radiation magneto gas dynamics for one dimensional flow are

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial r} + J \frac{\rho u}{r} = 0 ;$$

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{\partial h^2}{\partial r} = 0 ;$$

$$\frac{Dh}{Dt} + u \frac{\partial u}{\partial r} + \frac{\partial hu}{r} = 0 ;$$

$$\frac{DE}{Dt} + p \frac{D}{Dt}\left(\frac{1}{\rho}\right) + \frac{1}{\rho r^j} \frac{\partial}{\partial r}\left(F r^j\right) = 0$$

where

$$\frac{D}{Dt} = \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}\right)$$

where

$$P = E_m + E_r + E_h$$

$$E_m = P_m / r - 1$$

and F is the radiation heat flux, where suffixes m, R, h attached to the quantities to expressions for material radiation and magnetic terms respectively.

SIMILARITY PRINCIPLE AND SELF SIMILAR GAS MOTION

It is not always possible to solve non-linear differential equations describing the physics of a motion or a process by using mathematical techniques. An approximate solution of such a problem can be obtained by solving a similar problem which is easier to solve.

We consider the problem of one dimensional adiabatic flows of a perfect gas with constant specific heat, with either planer, cylindrical or spherical symmetry. The system of equation for flow of this type is given. These gas dynamic equations contain five dimensional quantities p, ρ, u, r and t. The dimensions of three of which are independent, for example density, distance

and time. So the equations admit three independent similarity transformation groups of quantities. Dimensional analysis can be used to obtain these groupings. By the successive application of these three groups we can obtain solution for the different flow similar to each other with altered density, length and time scales. Analogous transformations are made at the same time in the initial and boundary conditions of the problem.

The motion itself may be described by the most general functions of the two variables r and t, $\rho(r,t)$, $p(r,t)$ and $u(r,t)$. These functions also contain the parameters entering the initial and boundary conditions of the problem. They do not depend upon the position r and time t independently but are functions only of the combination (r / t). In other words, the distribution of all quantities with respect to r change with time without changing their form they remain similar to themselves. This type of motion in which the distribution of the flow variables remain similar to themselves, (i.e. similarity in the motion itself) with time and very only as a result of changes in scale is called self similar. Consider the distribution of pressure. The function p (r,t,) can be written in the form $p(r,t) = \pi(t)\, p(r/R)$, where $\pi(t)$ is the scale of the pressure and $R(t)$ is the length scale both depend on time in same manner, and the dimensionless ratio $p/\pi = p(r/R)$ is a function of new dimensionless coordinate $\eta = r/R$. Multiplying the variables p and η the scalar functions $\pi(t)$ and $R(t)$, we can obtain from the function $p(\eta)$, independent of time the true pressure distribution. The other flow variables, density and velocity, are expressed similarly.

It is a natural question that what requirements must be satisfied by the conditions of a problem in order that the motion be self similar.

Dimensional analysis is used to answer this question. Since the dimensions of pressure and density contain the unit of mass, at least one of the parameters in the problem must also contain a unit of mass. In many cases this is the constant initial density of the gas ρ_0, which has the dimension ML^{-3}. Let the parameter containing the unit of mass is a. It can be assumed that its

dimensions are $[a] = ML^k T^s$. The dimensions of the function p, ρ and u are $[p] = ML^{-1} T^{-2}$, $[\rho] = M L^{-3}$, and $[u] = LT^{-1}$, we can without any loss of generality represent them in the form suggested by Sedov.

$$P = \frac{a}{r^{k+1} t^{s+2}} p, \quad \rho = \frac{a}{r^{k+3} t^{s}} G, \quad u = \frac{r}{t} v$$

where p, G and V are dimensionless functions that depend on dimensionless groups containing r, t and the parameters of the problem.

For self similar motion it is possible to reduce a system of partial differential equations to a system of ordinary differential equations for new reduced functions of the similarity variable $\eta = r/R$, $R=R(t)$. The boundary and initial conditions of the problem are made dimensionless and in term transformed into conditions on the new unknown functions of η. This simplifies the problem greatly from the mathematical stand point and in a number of cases makes it possible to find exact analytic solutions.

The problem of a strong explosion represents a typical example of a self similar motion. This problem was formulated and solved by Sedov and succeeded in finding an exact analytic solution, to the equations of self similar motion. The same problem was also considered by Stanyukovich in his dissertation and by Taylor, both of whom formulated the equations for the problem and obtained numerical and not analytic solutions.

The parameters in the problem of a strong explosion are the initial density of the gas $\rho_o \sim ML^{-3}$ and the energy of explosion $E \sim ML^2T^{-2}$. The energy E is always equal to the total energy of the moving gas, and as a result an energy integral appears in the problem. These two parameters can not be combined to yield scalar with dimensions at either length or time. Hence the motion will be self similar that is, will be function of a particular combination of the coordinate r(distance from the center of the explosion) and the time t. The

initial pressure and speed of sound p and c in the problem of a strong explosion are assumed to be equal to zero, and hence these quantities are not parameters of the problem. So the quantity r/t can not serve as the similarity variable. In this case the only dimensional combination which contains only length and time is the ratio of E to ρ_o, with the dimension $[E/\rho_o] = L^5 T^{-2}$. Hence the dimensionless quantity

$$\eta = r \left(\frac{\rho_o}{E t^2} \right)^{1/5}$$

Can serve as the similarity variable. The distribution at pressure, density, and gas velocity can be expressed as functions of one dimensionless variable η.

CONCEPT OF SELF GRAVITATION

A fluid can be referred as self gravitating, when the fluid mass included is large and isolated and the gravitational attraction of other part of the fluid provides and volume force on any particular fluid element, for example, a gaseous star.

In the case of spherical symmetry the effect of all masses on a particle (point mass) at a distance r from the center of symmetry is equal to the force of attraction by the point mass placed at the center of symmetry and having a mass m (r,t).

The behavior of gravitationally interacting gaseous masses forming a star is given by the appropriate equations of motion.

The continuity equation for radial gas motion with spherical symmetry is

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left(\frac{\partial u}{\partial r} + j \frac{u}{r} \right) = 0$$

for the inviscid gas, momentum equation taking gravitational forces into account (according to Newton's Law of gravitation gravity g at r is $g=Gm/r^2$) can be written as

$$\frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{Gm}{r^2} = 0$$

where G is the gravitational constant and m (r,t) is the mass of gas with in the sphere of radius r.

To determine m we use the equation,

$$\frac{\partial m}{\partial r} = 4 \pi \rho r^2$$

which expresses the fact that the mass of a spherical shell of radius r and thickness ∂r is $4 \pi r^2 \rho \partial r$ and the energy equation for adiabatic gas motion.

$$\frac{\partial}{\partial t} \left(\frac{p}{\rho^\gamma} \right) + u \frac{\partial}{\partial r} \left(\frac{p}{\rho^\gamma} \right) = 0$$

These are the four equations with for unknowns p, ρ, V and m.

A very large no. of papers have been published by now in which analogous self similar solution were obtained to explain the adiabatic unsteady flow in self gravitating gas and analysed for systems of partial differential equations encountered in various problems of astrophysics such as internal motion in stars, motion of nebulae etc.

REFERENCES

1.	F. Chorlton	Text book of Fluid Dynamics D. Van. Nostrand Company ltd., London	(1957)
2.	R. Courant & Fredricks	Supersonic Flow and Shocks waves Insicience Publications Inc., New York	(1940)
3.	H. W. Emmons	Fundamentals of Gasdynamics, Princeton University Press, New Jersey	(1958)
4.	R.P. Kanwel	Magneto Hydrodynamics shocks waves J. Math Mech 9.681	(1960)
5.	W. J. M. Rankine	On theory of wave of finite longitudinal disturbances. Trans, of Royal Society of London, 160, 277	(1870)
6.	H. Hygoniot	Jr. d.e. Lolole Polytechnique 58, 1	(1889)
7.	Hoffman & Thylor	MHD Shocks, Phys Rev 80, 692,	(1950)
8.	W.Bleakney & Taub	Introduction of shockwaves, Rev. Mod Phys 21,58	(1949)
9.	R.G. Sacha	Phys, Rev, 69, 514	(1946)
10.	H.K. Sen & A. W. Gauss	Phys, Rev, 108, 560	(1957)
11.	R. E. Marshak	Phys, Fluid, 1,2,	(1958)
12.	K. C. Wang	J. Fluid Mech, 20, 447	(1984)

13. I. Von Neumann & R. D. Richtmyer — J. Appl, Phys, 21, 232 (1950)

14. P. L. Bhatnagar & P. L. Sachdeva — Nuobo – Cemento, 44, 15 (1966)

15. L. D. Landau & E. M. Lifshdtz — Electrodynamics of Continuous Media, Peragamon Press, New York (1960)

16. S. I. Pai — Magnetogasdynamist and Plasma Dynamics, Springer, verlo (1962)

17. G. B. Witham — J. Fluid Mechanics, 4 337 (1958)

18. YA. B. Zel. Dovichard & Raizer — Shockwaves and Radiation – An article in Annaul Review of fluid mechanics, 1, 385 (1959)

19. L. I. Sadov — Similarity and Dimentional Methods in Mechanics, Academic Press, New York (1959)

20. G. Deb Ray & S. N. Banerjee — Astrophysics & Space Science 68-73 (1980)

21. D. Sumener — Astron Astro Phys, 45, 151 (1975)

22. Phillip Posenan — Phys, Fluids 20, 1097 (1977)

23. J. B. Singh & PR — Astrophysics & Space Science, 87, 347 (1982)

24. Waiker & Taub — Reviewed modern physics vol. no. 4 page 585 (1949)

25. L. A. Elliot — Proc Royal Soc. 258A, 287 (1960)

26.	J. D. Jackson	Classical Electrodynamics, John Willy and Sons. Inc. Newyork	(1975)
27.	S. Chandrashekhar	An introduction of the study of stellar structure : Dover publication Inc., Newyork	(1957)
28.	C. J. Kynch	Modern developments in fluid dynamics, High speeds flow : clarendon oxford	(1953)
29.	G. I. Tayler	Proc. Roy. Soc. London : A 201, 159,	(1950)
30.	G. A. Liubinov	On possible kinds of one dimensional unsteady viscous gas motion, A paper is thermodynamics vol 7, Mascow.	
31.	L. I. Sedov	Compt. Rend (Doklady) Acad. Sci. USSR. 52, 17, Prinkl, Math, Mech. 10, 241	(1946)
32.	J. P. Billingsley	Comparision on experimental and predicted arial pressure variation for semi infinite metalic targets, AIAA paper 69-361	
33.	W. J. Jae	Anlytical studies of impact generated shock propagation survey and new results, R. Kinslaw, Acad. Press	(1970)
34.	J. Van Nevmann & R. D. Richtmyer	J. Appl. Phys. 21, 232	(1950)
35.	W. Chester	Phil Mag., 45, 1243	(1954)
36.	R. F. Chisnell	J Fluid Mech, 2, 286	(1957)

CHAPTER - 2

PROPAGATION OF PLANE RADIATIVE SHOCK WAVE IN UNSTABLE HOMOGENEOUS MEDIUM

INTRODUCTION

Scalar [2] and Pai [1] was the first person to investigate the problem of dynamical repture of the equilibrium of air masses in a non – radioactive state of a gas assuming that the velocity of an element of gas at a distance r at time t to be given by $U=\theta r/t$ where θ is constant. Deb Ray [3] has studied the Same problem. Carrus Etal [5] discussed an analytical solution of same assuming that medium at rest.

In this chapter we consider the disturbances headed by an radioactive shock surface of a rotating thunder in the earth medium and further we solve the problem of Deb-Ray [4]. Further we assume that through out the flow the temperature gradient is zero and we take specific heat index γ does not occur except that of term of internal heat energy. Which is a part of internal energy inside a thunder shock wave. In whole problem we take $\theta = 2/3$. As Deb Ray [4] in our problem also directly prove the result that mass enclosed within the surface of rotating shock wave at any time equals that under the same radius of a undisturbed state of the gaseous mass parcel of thunder system without imposing any special restriction on any of the parameters associated with the solution. The gaseous medium is taken to be in air.

SELF SIMILAR FORMULATION

The equation of motion governing the flow behind a shock wave are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} = 0 \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\partial}\frac{\partial p}{\partial r} + \frac{GM}{r^2} = 0 \qquad (2.2)$$

$$\frac{\partial T}{\partial r} = 0 \qquad (2.3)$$

Where u, ρ and p are the velocity density and pressure at the gas at a radial distance r from the origin at time t. m is the mass within a radius r and G the gravitational constant. T denotes the temperature. From equation (2.3) we get

$$p/\rho = \text{Constant} \qquad (2.4)$$

equation of continuity expressed as

$$\frac{\partial m}{\partial t} + u\frac{\partial m}{\partial r} = 0 \qquad (2.5)$$

The disturbance is headed by an isothermal shock and the condition are

$$\rho_1 (v-u_1) = \rho_o v \qquad (2.6)$$

$$p_1 - p_o = \rho_o v u_1 \qquad (2.7)$$

$$T_1 = T_o \qquad (2.8)$$

$$m_1 = m_o \qquad (2.9)$$

Where p_o, ρ_o, m_o and T_o are undisturbed pressure, density, mass and temperature just ahead of the shock and p_1, ρ_1 and u_1 are the quantities just behind the shock from equation 2.6 and 2.8 we get

$$u_1 = (1-\beta) v \qquad (2.10)$$

$$p_1 = \frac{1}{\beta} p_o \qquad (2.11)$$

$$\rho_1 = \frac{1}{\beta} \rho_0 \tag{2.12}$$

and

$$\beta = \frac{c^2}{V^2} \tag{2.13}$$

where $c^2 = p_o/\rho_o$ is the square of the sound velocity ahead of the shock.

Equation (2.6) shows that the mass particle behind the shock front moves with a velocity less than that of the shock front. Hence in the undisturbed state that mass confined with in the shock front at any time t must be equal to that contained within the surface of radius R.

Where R is the shock radius at time t.

i.e. $R = R(t)$

Thus we have

$$\int_0^R 4\pi\rho r^2\, dr = \int_0^R 4\pi\rho_o r^2\, dr \tag{2.14}$$

Equation (2.14) can be transformed to the nondimensional form

$$\int_0^1 \frac{\rho}{\rho_1} \varepsilon^2 \partial\varepsilon = \beta / (3-\mu)$$

Where we have taken $\varepsilon = r / R$

and $\rho_o = Vr^{-\mu}$ (2.15)

V being a constant

Since the stellar model of infinite radius is initially in hydrostatic equilibrium due to its self gravitation,
we have

$$p_0 = 2\,\pi\,G\,V^2\,r^{2-2\mu}\,/\,(\mu - 1)\,(3 - \mu)$$

for the positively of the equilibrium pressure $P_{o,}$ we should have

$$1 < \mu < 3 \qquad (.216)$$

let us seek a solution of equation (2.1) and (2.5) in the form

$$u = \theta \;\; \frac{r}{t} \qquad (2.17)$$

SIMILARITY SOLUTION

In order to reduce the equation of motion to ordinary equation we now introduce the following similarity transformation, let similarity variables ηbe in the form

$$\eta \quad = \quad r^a\, t^b$$

$$\rho \quad = \quad r^k\, t^\lambda\, \Omega\,(\eta) \qquad (2.18)$$

$$p \quad = \quad r^{k+2}\, t^{\lambda-2}\, p\,(\eta) \qquad (2.19)$$

$$m \quad = \quad r^{k+3}\, t^\lambda\, z\,(\eta) \qquad (2.20)$$

The constant K, λ, a, and b are to be determined form the condition of the problem.

The total energy Q inside a shock wave of radius R is given by

$$Q = 4\pi \int_0^R \left(\frac{1}{2}\rho u^2 + \frac{p}{(\gamma - 1)} - \frac{Gm\rho}{\gamma}\right) r^2 dr \qquad (2.21)$$

Where as the total energy with in the same volume in the equilibrium state is given by

$$H = 4\pi \int_0^R \left(\frac{\rho_o}{\gamma - 1} - \frac{Gm_o\rho_o}{\gamma}\right) r^2 dr \qquad (2.22)$$

Where

$$m_0 = 4\pi \int_o^R \rho_o r^2 \, dr = \frac{4\pi v r^{(3-\mu)}}{3 - \mu} \qquad (2.23)$$

Since the motion is due to a straight excitation of an unstable condition of air mass we have

$$Q = H \qquad (2.24)$$

We consider the shock surface to the given η by η_0 = constant

Where η_0 is the shock front Value of η

We have $V = - bR/at$ (2.25)

Let us consider K=0, $\lambda = -2$ substituting equation (2.17)-(2.19) in the equation (2.10) - (2.12) and solving for Q and ρ (η_o) and (η_o), we get

$$\theta = - \frac{b}{a(1-\beta)} \qquad (2.26)$$

$$p(\eta_0) = \frac{1}{\beta\,[2\,\pi\,G\,v^2 / (\mu - 1)\,(3 - \mu)\,]\,R^{-2\mu}\,t^{u}} \qquad (2.27)$$

$$\Omega(\eta_0) = \frac{V}{\beta\,R^{-\mu}\,t^2} \qquad (2.28)$$

let β at the shock defined by

$$\beta = \frac{C_o^2}{V^2} = \frac{a^2}{b^2}\,R^{-\mu}\,t^{-t}\,2\,\pi\,G\,v / (\mu - 1)\,(3 - \mu) \qquad (2.29)$$

the right hand side of equation (2.27) ---- (2.29) are function of η_0 if

$$-\frac{\mu}{2} = \frac{a}{b} \qquad (2.30)$$

thus we have without any loss of generality

$$k=0,\ \lambda = -2,\ a = -\alpha,\ b = 2 \qquad (2.31)$$

hence we obtain from (2.26)

$$0 = \frac{2}{\mu(1-\beta)} \qquad (2.32)$$

which express the value of 0 in terms of μ & β also from (2.30)

we have

$$\beta = \mu^2\,\pi\,G\,v\,\eta_o / 2\,(\mu - 1)\,(3 - \mu) \qquad (2.33)$$

which is a constant

The following four ordinary differential equation are derived from equation (2.1 - 2.5)

In terms of η by using (2.17 2.21) and (2.31)

$$- \theta + \theta^2 + 1/\Omega (\eta) [2 p (\eta) - \alpha \eta p (\eta)] + G Z(\eta) = 0 \tag{2.34}$$

$$p^1 (\eta) / p (\eta) = [4 + 2\mu - 5 \mu \theta / \mu (2 - \mu\theta)]1/ \eta \tag{2.35}$$

$$\Omega^1 \eta / \Omega (\eta) = [(2- 3\theta) / (2 - \mu \theta)] 1/\eta \tag{2.36}$$

$$Z1 (\eta) / Z (\eta) = (2 - 3\theta) / 2 - \mu \theta)] 1/\eta \tag{2.37}$$

After integrating (2.36 – 2.37) with respect to η we obtain the following result

$$p = p_1(r/R)^{2 - \mu (2 - 3\theta / 2 - \mu \theta)} \tag{2.38}$$

$$\rho = \rho_1(r/R)^{-\mu (2-3\theta / 2-\mu \theta)} \tag{2.39}$$

$$m = m_1(r/R)^{(6-2\mu) / (2 - \mu \theta)} \tag{2.40}$$

where m_1 denotes the mass within the shock surface equation (39) we get

$$\int_0^1 \frac{\rho}{\rho_1} \xi^2 d\xi = (2 - \mu \theta) / 2 (3 - \mu) \tag{2.41}$$

where as by (32) we get

$$\beta / 3 - \mu = (2 - \mu \theta) / 2 (3 - \mu) \tag{2.42}$$

So we find from equation (2.41) and (2.42) that equation (2.13) is identical satisfied for all constant values of θ, μ and β subject to the shock condition (2.32) from (2.36) we also have.

$$P(\eta) = P (\eta_o) (\eta / \eta_o)^{[(4 + 2 \mu - 5 \alpha \theta) / (2 - \mu \theta) \mu]}$$

$$\Omega(\eta) = \Omega (\eta_o) (\eta / \eta_o)^{[(2- 3 \theta) / (2 - \mu \theta)]}$$

$$Z (\eta) = Z (\eta_o) (\eta / \eta_o)^{[(2- 3 \theta) / (2 - \mu \theta)]}$$

Using these values in (33) and (34) we get

$$- \theta^2 + \theta^2 + 4 \beta (3 \theta - 2) / \mu (2 - \mu\theta) (\eta / \eta_o)^{v / \mu} +$$

$$8(\mu-1)\,\beta / \mu^2 \,(\eta / \eta_0)^{[(2-3\theta)/(2-\mu\theta)]} = 0 \qquad (2.43)$$

Equation (2.43) is to be also identically satisfy for all value of η hence the terms involving η either constant or zero.

One set of solution is given by

$$\theta = 2/3 \qquad (2.44)$$

from (33) by using the value of above value of θ we get

$$\beta = (3-\mu)/3 \qquad (2.45)$$

hence from (2.43) we have

$$\beta\,\mu^2 - 48\,\mu + 36 = 0 \qquad (2.46)$$

We have on simplification

$$\mu = 2.60 \quad \text{or} \quad \mu = 1.04 \qquad (2.47)$$

both these values lies within the range given by equation (2.16) using above values of μ the equation (2.45).

We get

$$\beta = 0.13 \quad \text{or} \quad \beta = 0.65 \qquad (2.48)$$

using the values for r, ρ, p and m also for ρ_0 P_0 and m_0 we get from (2.25)

$\theta^2 \mu^2 / 4\beta(\mu-1)(3-\mu)\,[(2-\mu\theta)/(10-2\mu\theta-2\mu)] + (2-\mu\theta)/(2\beta(\gamma-1)(\mu-1)(3-\mu)^2)$

$-2(2-\mu\theta)/\beta(3-\mu)(10-4\mu+\mu\theta) = [(2\gamma-1-2\mu(\gamma-1)]/(\gamma-1)(\mu-1)(3-\mu)(5-2\mu)$

we have $\mu \neq 5/2$ and $\mu \neq 3$, $\mu \neq 1$ (2.49)

CONCLUSION

We find that the two possible sets of solutions given by

$\theta = 2/3$, $\mu = 2.60$, $\beta = 0.13$ and

$\theta = 2/3$, $\mu = 1.04$, $\beta = 0.65$

where γ has its independent value the equation satisfy by substitution in (2.49) the numerical values are different from the case of non radioactive besides these two sets there is no other possible solution of this two type in such system. Viz. from θ = constant bearing any physical significance the value of θ i.e. 2/3 is the same for both the cases radioactive and non – radioactive.

These two sets of solution are interesting one in which the pressure and density and uniform over that whole mass at any instant of time t (given). But in radioactive expansion cases the same fall in density as well as in pressure throughout the air mass of a sudden expansion in the atmosphere in the homogenous medium of gaseous is in an radioactive state as ρ / p = constant or $\rho \propto p$ (ρ is directly proportional to pressure).

These phenomenon are normally helpful in dealing with the astrophysical problems on account of clouds condition produce the plane shocks in the compressible gases of atmosphere. These are strong shocks which propagate with high mach number and dissipates in the lower strata of atmosphere on account of viscous layer prevailing.

REFERENCES

1.	Pai S. I.	:	Introduction to theory of compressible flow D Van Nostrand Company in C, New York.		(1959)
2.	Scalar B. I.	:	Doke Akad Nauk, USSR	112, 211	(1957)
3.	Deb-Ray	:	Astrophy Space Science	47, 229	(1977)
4.	Deb-Ray	:	Bull Cal. Math, SOC	609,225	(1975)
5.	Carrus Etal	:	Astrophy, Jr.	113, 193	(1991)
6.	Rosenou P	:	Phys, Fluids	20,1097	(1977)

CHAPTER – 3

ANALYTICAL SOLUTION OF SELF GRAVITATING GAS BEHIND A CYLINDRICAL SHOCK WAVE IN MAGNETO GAS DYNAMICS

INTRODUCTION

The unsteady motion of a large mass of gas followed by sudden release of energy result in novae and supernovae. The explanation and analysis of the internal motion in stars in one of the basic problems in astrophysics. Numerical solution for self similar adiabatic flow in self gravitating gas were obtained by Carrus etal at [1] and Sedov [3] starting with these results. Ryazanov [2] obtained a particular analytic solution. But this solution does not describe the flow behavior in general. Singh [4] has discussed the self similar adiabatic flow of self gravitating gas in ordinary gas dynamics and has obtained numerical solution.

In this chapter we show that the azimuthal magnetic filed has a significant effect on physical parameters when the gas is self gravitating. The motion of the shock wave is assumed to satisfy the power low.

$$\rho_0 \quad = \quad A\,R^{-\alpha} \qquad (3.1)$$

where A and α are constants and R is the shock radius. The total energy of the flow increases with time because of the pressure exerted on the gas by an expanding surface.

Therefore,

$$E \quad = \quad B\,t^{q} \qquad (q \geq 0) \qquad (3.2)$$

where B and q are constants and E is the total energy. The magnetic field distribution low is

$$h_o = CR^{-\beta} \qquad (\beta \geq 0) \tag{3.3}$$

where C and β are constant and the value of q and β to be determined later. The magnetic field is directed tangential to the advancing shock front.

The flow variable just ahead of the shock are

$$u_o = 0, \qquad m_o = \frac{4\pi A}{3-\alpha} R^{3-\alpha}$$

and

$$\rho_o = \frac{2\pi A^2 G}{(\alpha-1)(3-\alpha)} R^{2-2\alpha} + \frac{C^2}{2\beta}(1-\beta)R^{-2\beta} \tag{3.4}$$

where

$$1 + \beta = \alpha \tag{3.5}$$

and viscosity is neglected.

We investigate three type of models, the first having total energy of the explosion to be constant ($\alpha = 2.5$), the second having constant velocity of propagation of shock waves ($\alpha = 2$) and the third having neither constant total energy nor constant velocity propagation of shock wave ($\alpha = 1.5$). An idealized magnetic field is considered for only a portion of sphere enclosing the origin i.e. the point of explosion.

EQUATION OF MOTION AND BOUNDARY CONDITION

The equation governing the cylindrically symmetrically flow of rotating gas under the influence of transverse magnetic field are written as.

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\partial u}{r} = 0 \qquad (3.6)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{(H_0 + H_z)}{\rho} \frac{\partial (H_0 + H_z)}{\partial r}$$

$$+ \frac{(H0 + Hz)^2}{\rho r} + \frac{Gm}{r^2} = 0 \qquad (3.7)$$

$$\frac{\partial (H_0 + H_2)}{\partial t} + u \frac{\partial (H_0 + H_z)}{\partial r} + (H_0 + H_z) \frac{\partial u}{\partial r} + \frac{(H_0 + H_z)u}{r} = 0 \qquad (3.8)$$

$$\frac{\partial}{\partial t} (p \rho^{-\gamma}) + u \frac{\partial}{\partial r} (p \rho^{-\gamma}) = 0 \qquad (3.9)$$

$$\frac{\partial m}{\partial r} = 4 \pi r^2 \rho \qquad (3.10)$$

where r, t, m, u, ρ, p and γ are radial distance from the centre, time, mass contained in a sphere of radius γ, velocity, density, pressure and ratio of two specific heats, respectively, as G represents the gravitational constant.

The Rankine – Hungoniot boundary conditions at the shock (3.10) are

$$u_1 = \frac{\varepsilon - 1}{\varepsilon} V \qquad (3.11)$$

$$\rho_1 = \rho_o \rho \qquad (3.12)$$

$$p_1 = \psi \rho_o V^2 \qquad (3.13)$$

$$H_1 = H_0 \rho \qquad (3.14)$$

$$m_1 = m_0 \qquad (3.15)$$

where

$$\psi = \frac{1}{\gamma M^2} + \frac{2(\varepsilon - 1)}{[(\gamma+1) - (\gamma-1)\ ]}\left[\frac{1}{M^2} + \frac{\gamma - 1}{4M^2}(\varepsilon - 1)^2\right] \qquad (3.16)$$

$v = \dfrac{dR}{dt}$ being the shock velocity and is give by the quadratic equation

$$M_A^{-2}(2 - \bar{\alpha})\,\varepsilon^2 + \left[(2\gamma - 1) + \frac{2}{M^2}\right]\varepsilon - (\gamma + 1) = 0 \qquad (3.17)$$

The Mach number M and Alfven's Mach number M_A are given by

$$M^2 = \frac{V^2 \rho_o}{\gamma\, p_o} \text{ and } M_A^2 = \frac{V^2 \rho_o}{H_o^2} \qquad (3.18)$$

let us seek the solution to the equation in the following form

$$u = \frac{r}{t}\, u(\eta), \qquad \rho = r^k t^\lambda\, \Omega(\eta)$$

$$m = r^{k+3}\, t^{\lambda}\, w(\eta) \qquad (3.19)$$

$$p = r^{k+2}\, t^{\lambda-2}\, p(\eta),\ H = H_0 = H_z = r^{(k+2)/2}\, t^{(\lambda-2)/2}\, N(\eta) \qquad (3.20)$$

where $\eta = r^a t^b$; while k, λ, a and b are constants and are to be determined from the condition of the problem. The total energy E inside the shock wave of radius R is given by

$$E = 4\pi \int_{o}^{R} \left(\frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1} + \frac{h^2}{2} - \frac{Gm\rho}{r} \right) r^2 \, dr = B t^q \ (q \geq 0)$$

(3.21)

In terms of the variable η, we can express the total energy as

$$E = \frac{4\pi}{a} \int_{\eta}^{\eta_o} [n^{(k+5)/a]-1} t^{\lambda - 2 - (b/a)(k+5)} \left(\frac{1}{2} U^2 \Omega + \frac{p}{\lambda - 1} + N^2 \right)$$

$$- G w \Omega \eta [^{(2k+5)/a]-1} t^{2\lambda - (b/a)(2k+5)}] \, d\eta = B t^q \qquad (3.22)$$

η_o and η being the values of η at the shock front ad expanding surface, respectively

Equation (3.22) yields

$$\frac{a}{b} = \frac{-5}{4+q} \qquad (3.23)$$

we choose η_o to be constant at the shock surface. This choice fixes the velocity of the shock as.

$$V = - \frac{b}{a} \frac{R}{t} \qquad (3.24)$$

using equation (3.19) and (3.23) in equations (3.11) – (3.14) and in equation (3.18) we assume, without any loss of generality.

$$K = -\alpha, \qquad \lambda = 0, \quad b = 4 + q, \quad a = -5 \text{ and } \alpha = \frac{10}{4+q}$$

(3.25)

Hence

$$R = \eta_o^{-1/5}\, t^{(4+q)/5} \tag{3.26}$$

where it is assumed that initially R = 0 from equation (3.26) it can be seen that the value q = 1 corresponds to uniform expansion at the sphere.

ANALYTICAL SOLUTION OF THE PROBLEM

$$5\eta \frac{\overline{\Omega}'}{\overline{\Omega}}\left(\frac{1}{\alpha} - u\right) + (2-\alpha)u - 5\eta u' = 0 \tag{3.27}$$

$$5u'\eta\left(\frac{1}{\alpha} - u\right) + u(u-1) + \frac{\overline{p}}{\overline{\Omega}}(1-\alpha) - 5\eta\frac{\overline{p}'}{\Omega} - \frac{\overline{N}^2}{\Omega}\left(\frac{\alpha-3}{\alpha} + 5\eta\frac{\overline{N}'}{\overline{N}}\right) +$$

$$\frac{(2-\alpha)(\alpha-1)}{\pi\alpha^2}\,\frac{1}{\eta^2}\,\overline{LW} = 0 \tag{3.28}$$

$$5\eta\frac{\overline{N}'}{\overline{N}}\left(\frac{1}{\alpha} - u\right) + u - 5\eta u' + \frac{u-1}{1} - \alpha u = 0 \tag{3.29}$$

$$5\eta\frac{\overline{P}'}{\overline{p}}\left(\frac{1}{\alpha} - u\right) + u(1-\alpha) - 1 - \gamma\left[-\alpha u + 5\eta\frac{\overline{\Omega}'}{\overline{\Omega}}\left(\frac{1}{\alpha} - u\right)\right] = 0$$

(3.30)

$$5\eta\frac{\overline{w}'}{\overline{w}} + 4\pi\frac{\overline{\Omega}}{\overline{w}} - (2-\alpha) = 0 \tag{3.31}$$

where

$$\bar{\Omega}^{-1} = \frac{\bar{\Omega}'}{A}, \quad \bar{P}' = \frac{P'}{A}, \quad \bar{N}' = \frac{N'}{A}, \quad \bar{W}' = \frac{W'}{A} \tag{3.32}$$

$$\bar{\Omega} = \frac{\Omega}{A}, \quad \bar{P} = \frac{P'}{A}, \quad \bar{N} = \frac{N}{A}, \quad \bar{W} = \frac{W}{A}$$

$$L = \frac{1}{\gamma M^2} - \frac{1-\alpha}{1(\alpha - 1) M_A^{+1}} \tag{3.33}$$

and prime denotes the differentiation with respect to η. The jump conditions at the shock are given by

$$\bar{U} = (\eta_o) = \frac{1}{\alpha} \quad \frac{\varepsilon - 1}{\varepsilon}, \qquad \bar{\Omega}(\eta_o) = \varepsilon, \bar{p}(\eta_o) = \frac{3}{\alpha^2}\psi \tag{3.34}$$

$$\bar{N}(\eta_o) = \frac{2\varepsilon}{\alpha} M_A^{-1} \qquad \text{and } \bar{W}(\eta_o) = \frac{4\pi}{2-\alpha}$$

RESULT AND DISCUSSION

Similarity solution of the problem of propagation of a spherical magneto gas dynamic shock wave have been obtained. For numerical calculation the flow and field variables have been taken in the following non dimensional forms.

$$\frac{u}{u_1} = \left(\frac{\eta}{\eta_o}\right)^{1/a} \qquad \frac{u(\eta)}{u(\eta_o)} \tag{3.35}$$

$$\frac{\rho}{\rho_1} = \left(\frac{\eta}{\eta_o} \right)^{-\alpha / a} \frac{\Omega(\eta)}{u(\eta_o)} \tag{3.36}$$

$$\frac{p}{p_1} = \left(\frac{\eta}{\eta_o} \right)^{(1 - \alpha) / a} \frac{p(\eta)}{p(\eta_o)} \tag{3.37}$$

$$\frac{H}{H_1} = \left(\frac{\eta}{\eta_o} \right)^{(1 - \alpha)\, \alpha} \frac{N(\eta)}{N(\eta_o)} \tag{3.38}$$

and

$$\frac{m}{m_1} = \left(\frac{\eta}{\eta_o} \right)^{(2 - \alpha) a} \frac{W(\eta)}{W(\eta_o)} \tag{3.39}$$

REFERENCES

1. Carrus. P. Fox. P. Hass, F, and Kopal, Z (1951)
   Astrophy J. 125,495

2. Ryazanov, E. V., Sovier Phys, Doki SSSR 4,537 (1959)

3. Sedov, L. I. Similarity and Dimensional methods in mechanics, Academic Press, New York. (1959)

4. Singh J B Astrophys Space Sci 87, 347 (1982)

5. Whitham, G B.; Fluid Mech 4, 337 (1959)

CHAPTER – 4

ANALYTICAL SOLUTION OF SPHERICAL SHOCK WAVE IN A ROTATING GAS WITH AXIAL COMPONENT OF MAGNETIC FIELD

INTRODUCTION

The propagation of hydromagnetic shock through a rotating gas being close to the actual situation of thunder. Kumar and Prakash [1,2,3] recently using C.C.W. [4,5,6] method have investigated the propagation of weak and strong diverging cylindrical hydromagnetic gas taking density distribution variable but axial component of magnetic field of constant strength.

In this chapter, using the C.C.W. method we have considered the problems of the propagation of a converging and diverging spherical shock wave through a rotating gas under the influence of a magnetic field of constant axial, simultaneously for both weak and strong waves we have assumed initial distribution is constant.

The analytical expression for shock velocity and shock strength have been obtained for weak shock under the two conditions namely when the magnetic field is weak and when the magnetic field is strong. For strong shock also we have considered two cases, i.e. when the magnetic field is strong and when ρ_o/p_1 is approximately equal to $\gamma+1/\gamma-1$ which is purely a non magnetic case.

BASIC EQUATIONS BOUNDARY CONDITION AND AN ANALYTIC EXPRESSION FOR SHOCK VELOCITY

The equation governing the spherical symmetrical flow of rotating gas under the influence of axial component of magnetic fields are written as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2}{r} + \frac{\mu}{2\rho}\frac{\partial}{\partial r}\frac{H_z^2}{2\rho} = 0 \qquad (4.1)$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(v\,r) = 0 \qquad (4.2)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (4.3)$$

$$\frac{\partial H_z}{\partial t} + \frac{\partial H_z}{\partial r} + H_z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0 \qquad (4.4)$$

where u, p and ρ are the velocity, pressure and density behind the shock front respectively while H_z is axial components of field H.

The [illegible] of a single parameter.

$$N = \rho_1/\rho_o \quad \text{as}$$

$$\rho_1 = \rho_o N, \qquad H_1 = H_o N, \qquad u_1 = \frac{N-1}{N}U$$

$$U^2 = \frac{2N}{(\gamma+1)-(\gamma-1)N}\left[a_o^2 + \frac{b_o^2}{2}\{(2-\gamma)N+\gamma\}\right]$$

$$p_1 = p_o + \frac{2\rho_o(N-1)}{(\gamma+1)-(\gamma-1)N}\left[a_o^2 + \frac{\gamma-1}{4}b_o^2(N-1)^2\right] \qquad (4.5)$$

where 0 and 1 denote respectively the states immediately ahead and behind the shock front; U the shock velocity, a the sound speed ($\gamma\ p_o/\rho_o$) and the Alfven speed is ($\mu\ H_o^2\ /\ \rho_o$)

where

$$H_o^2 = Hz_o^2$$

In the equilibrium state we have

$$\frac{1}{\rho_o}\ \frac{d\rho_o}{dr} - \frac{V^2}{r} = 0$$

Integrating the proceeding equation we get.

$$P = K_1 \log \frac{A}{r} \text{ and } a_o = K_2 \left[\log \frac{A}{r}\right]^{1/2}$$

where $K_1 = \rho' V^2$

$$K_2 = \left(\frac{\gamma K_1}{\rho'}\right)^{1/2} \quad \text{and}$$

$\rho'\ V^2 \log A$ is constant of integration.

WEAK SHOCKS

For a very weak shock we take the parameter N as

$$\frac{\rho}{\rho_o} = N = 1 + \xi, \qquad \xi << 1$$

Now consider the two cases of weak and strong magnetic fields.

Case I :

When the magnetic field is weak i.e.

$b_o^2 << a_o^2$ (i.e. $\mu H_o^2 << \gamma p_o$)

and therefore, the boundary conditions [4.2] reduce to

$\rho_1 = \rho_o (1 + \varepsilon)$

$u_1 = b_o \varepsilon$

$Hz_1 = Hz_o (1 + \varepsilon)$

$P1 = p_o (1 + \gamma \varepsilon)$

$$U = \left(1 + \frac{\gamma + 1}{4} \varepsilon \right) a_o \tag{4.6}$$

Case 2 :

When magnetic field is strong $b_o^2 >> a_o^2$ under these circumstances the boundary condition (4.2) reduce to

$\rho_1 = \rho_o (1 + \varepsilon)$

$u_1 = b_o \varepsilon$

$Hz_1 = Hz_o (1 + \varepsilon)$

$p_1 = p_o (1 + \varepsilon)$

$$U = \left(1 + \frac{1}{4}\gamma\right) b_0 \qquad (4.7)$$

STRONG SHOCK

Case I

When the magnetic field is strong $b_0^2 >> a_0^2$ (i.e. $\mu Ho^2 >> \gamma p_o$) under this condition the boundary condition (4.2) becomes.

$$p_1 = p_o N$$

$$H_1 = H_o N$$

$$U_1 = \frac{N - 1}{N} U$$

$$\frac{p_1}{p_o} = \frac{XU^2}{a_0^2} + L$$

where

$$X = \frac{\gamma (\gamma - 1) (N - 1)^3}{2N \{(2 - \gamma) N + \gamma\}} \quad \text{and}$$

$$L = \frac{(\gamma + 1) N - (\gamma - 1)}{(\gamma + 1) - (\gamma - 1)N} \quad \text{and} \qquad (4.8)$$

Case II

For strong shock when $b_0^2 << a_0^2$ (i.e. when $N \rightarrow [\,(\gamma+1)/(\gamma-1)]$ is small the field may be regarded as independent of magnetic field.

For converging shock the characteristics form of system of equation (4.2) is

$$dp + \mu\,(H_z\, dH_z) - \rho c\, du + \left[\frac{\rho c^2 u}{u-c} + \frac{\rho c v^2}{u-c}\right]\frac{dr}{r} = 0 \qquad (4.9)$$

where as the characteristic form for deverging shock is

$$dp + \mu\,(H_z\, dH_z) + \rho c\, du + \left[\frac{\rho c^2 u}{u+c} - \frac{\rho c v^2}{u+c}\right]\frac{dr}{r} = 0 \qquad (4.10)$$

where $c^2 = a^2 + b^2 = \dfrac{\gamma p}{\rho} + \dfrac{\mu H_z^2}{\rho}$

and $H_o^2 = H_z^2$

WEAK SHOCK WITH WEAK MAGNETIC FIELD

Substituting shock condition (4.6) into (4.9) and neglecting the second and higher order term of ξ, since $\xi << 1$, we get.

$$\left\{\frac{\mu}{\gamma p_o}\,(H_z^2{}_o)\right\} d \;+\left\{\frac{dp_o}{p_o} - \frac{da_o}{a_o} - \frac{dr}{r} - \frac{2dp_o}{\gamma p_o}\right\} = 0 \qquad (4.11)$$

Now substituting

$$\frac{dp_o}{p_o}, \quad \frac{da_o}{a_o}, \quad \text{and} \quad \frac{\mu Hz^2{}_o}{\gamma \rho_o}$$

in equation (4.11) we get

$$\frac{d\varepsilon}{\varepsilon} = \left[\frac{[\gamma - 4]}{\gamma \beta_1^2} + \frac{1}{\beta_1^2} \log \frac{A}{r} \right] \frac{dr}{r} \tag{4.12}$$

$$\text{where } \beta_1^2 = \frac{\mu H_{zo}^2}{\gamma k_1}$$

$$\beta_2^2 = 0$$

Integrating of equation (4.12) yields

$$\varepsilon = \overline{K}_3 \, r^{[\gamma - 4]/\gamma \beta_1^2} \exp \left\{ - \frac{1}{2\beta_1^2} \left(\log \frac{A}{r} \right)^2 \right\} \tag{4.13a}$$

where log K_3 is constant of integration in case of diverging weak shock with weak magnetic field, we have the relation

$$\varepsilon = K_3 \, r^{-1/2} \left(\log \frac{A}{r} \right)^{-q_1} \exp \left\{ - \frac{3}{8} \beta_1^2 \left(\log \frac{A}{r} \right)^{-1} \right\} \tag{4.13b}$$

where log K_3 is constant of integration and

$$q_1 = \left(\frac{3}{4} + \frac{\beta_1^2}{4} \right)$$

Hence from equation (13a) and (13b) we have

$$\frac{u}{a_0} = 1 + \frac{\gamma+1}{4} \overrightarrow{K_3}\, r^{[\gamma-(1-4\beta_1 2)-4]/\gamma\beta_1^2} \exp\{ -\frac{1}{2\beta_1^2} \left(\log \frac{A}{r} \right)^2 \qquad (4.14)$$

$$\frac{u}{a_0} = 1 + \frac{\gamma+1}{4} K_3\, r^{-1/2} \left(\log \frac{A}{r} \right)^{-q_1} \exp\{ - \frac{3}{8} \beta_1^2 \left(\log \frac{A}{r} \right)^{-1} \qquad (4.15)$$

for converging and diverging shock respectively.

WEAK SHOCK WITH STRONG MAGNETIC FIELD

Substituting the shock conditions (4.7) in (4.9) we get

$$d\varepsilon + \left\{ \frac{dp_o}{p_o} - \frac{\mu H_o^2}{\gamma p_o} \left(\frac{dr}{r} + \frac{db_o}{b_o} \right) - \frac{2dp_o}{\gamma p_o} \right\} = 0 \qquad (4.16)$$

Substituting the respective quantities in equation (4.16) we get

$$\frac{d\varepsilon}{\varepsilon} = \left[\frac{\gamma(1+\beta_1^2-2)}{\gamma} \right] \left(\log \frac{A}{r} \right)^{-1} \frac{dr}{r} \qquad (4.17)$$

integrating equation (4.17) we get

$$\xi = K_4 \left(\log \frac{A}{r} \right)^{-q_2} \tag{4.18a}$$

where log $\overline{K}_4$ is constant of integration and

$$q_2 = \frac{[\gamma (1 + \beta_1^2) - 2}{\gamma}$$

when we consider diverging shock with strong magnetic field we get

$$\xi = K_4 \log \left(\frac{A}{r}\right)^{-\beta_1^2 - 1} \exp \left\{ - 2\beta_1^2 (1 - \beta_1^2) \left(\log \frac{A}{r}\right)^{-1} \right\} \tag{4.18b}$$

where log $\overline{K}_4$ is constant of integration equation (4.18a) and (4.18b) disclose that

$$\frac{u}{b_0} = 1 + \frac{3}{4} \vec{K}_4 \left(\log \frac{A}{r}\right)^{-q_2} \tag{4.19}$$

$$\frac{u}{a_0} = 1 + \frac{3}{4} K_4 \left(\log \frac{A}{r}\right)^{\beta_2^2 - 1} \exp \left\{ - 2\beta_1^2 (1 - \beta_1^2) \left(\log \frac{A}{r}\right)^{-1} \right\} \tag{4.20}$$

for converting and driverging shocks respectively.

STRONG SHOCK

Substituting shock condition (4.8) into (4.9) we get

$$dU^2\left[\frac{x}{y} - \frac{N-1}{2}\left(\frac{X}{N}\right)^{1/2} + U^2\left[\frac{X}{Y}\quad\frac{dp_o}{p_o} - \frac{2x}{y}\quad\frac{da_o}{a_o} + \frac{x(N-1)}{(N-1)-\sqrt{N}x}\quad\frac{dr}{r}\right]\right.$$

$$+\left[L\frac{dp_o}{p_o} - \frac{\gamma k_1 N^2 \beta_1^2}{p_o}\left\{\frac{(N-1)+\sqrt{N}x}{(N-1)-\sqrt{N}x}\right\}\frac{dr}{r} + \frac{N\sqrt{N}x}{[(N-1)-\sqrt{N}x]}\right\} = 0 \qquad (4.21)$$

Substituting the respective values we get.

$$\frac{du^2}{dr} + \frac{B}{r}U^2 - cr^{-1} = 0 \qquad (4.22)$$

where $B = B'/M$

$C = C'/M$

$$B' = \frac{X(N-1)}{(N-1)\sqrt{XN}}$$

$$M = \frac{x}{y} - \frac{(N-1)}{2}\left(\frac{x}{N}\right)^{1/2}$$

and

$$C' = \frac{Lk_1}{\rho'} + \frac{\gamma k_1 N^2 \beta_1^2}{\rho'}\left\{\frac{(N-1)+\sqrt{NX}}{(N-1)-\sqrt{NX}}\right\} - \frac{N\sqrt{NX}}{(N-1)-\sqrt{NX}}$$

Integration of (4.22) yields

$$U^2 = \overline{K}_5 r^{-\beta} + \frac{C}{B} \tag{4.23}$$

where $\overline{K}_5$ is constant of integration. In the case of diverging strong shock with strong magnetic field we have

$$U_2 = K_5 r^{-\beta} + \frac{C}{B} \tag{4.24}$$

Where K_5 is constant of integration finally, for converging and diverging shock we have

$$\frac{U^2}{a_o^2} = \frac{1}{K_2^2 \log \frac{A}{r}} \left(\overline{K}_5 r^{-B} + \frac{C}{B}\right) \tag{4.25}$$

and

$$\frac{U^2}{a_o^2} = \frac{1}{K_2^2 \log \frac{A}{r}} \left(K_5 r^{-B} + \frac{C}{B}\right) \tag{4.26}$$

respectively.

In the last, the expressions for the velocity, the density, and the pressure of the gas just behind the shock surface are

$$U = K_2\overline{K_3}\,(\log \frac{A}{r})^{1/2}\, r^{(\gamma-4)/\gamma\beta_1^2} \exp\{-\frac{1}{2\beta_1^2}\log(\frac{A}{r})^2\} \tag{4.27a}$$

$$U = K_2K_3\, r^{-1/2}(\log \frac{A}{r})^{(1-2q_1)} \exp\{-\frac{3}{8}\beta_1^2 \log(\frac{A}{r})^{-1}\} \tag{4.27b}$$

$$\rho = \rho'[\,1 + \overline{K_3}\, r^{(\gamma-4)/\gamma\beta_1^2} \exp\{-\frac{1}{2\beta_1^2}(\log \frac{A}{r})^2 \tag{4.28a}$$

$$\rho = \rho'[\,1 + K_3\, r^{-1/2}(\log \frac{A}{r})^{-q_1}\exp\{-\frac{3}{8}\beta_1^2(\log\frac{A}{r})^{-1} \tag{4.28b}$$

$$p = k_1(\log\frac{A}{r} + \gamma k_3\, r^{(\gamma-4)/\gamma\beta_1^2}(\log\frac{A}{r})\exp\{-\frac{1}{2\beta_1^2}(\log\frac{A}{r})^2\} \tag{4.29a}$$

$$p = k_1(\log\frac{A}{r} + \gamma k_3\, r^{-1/2}(\log\frac{A}{r})^{1-q_1}\exp\{-\frac{3}{8}\beta_1^2(\log\frac{A}{r})^{-1}\} \tag{4.29b}$$

$$U = \beta_1 K_2 \overline{K}_4 \left(\log \frac{A}{r} \right)^{-q_2} \tag{4.30a}$$

$$U = \beta_1 K_2 \overline{K}_4 \left(\log \frac{A}{r} \right)^{\beta_1^2 - 1} \exp \left\{ - 2\beta_1^2 (1 - \beta_1^2) \left(\log \frac{A}{r}\right)^{-1} \right\} \tag{4.30b}$$

$$\rho = \rho' \left[1 + \overline{K}_4 \left(\log \frac{A}{r}\right)^{-q_2} \right] \tag{4.31a}$$

$$\rho = \rho' \left[1 + K_4 \left(\log \frac{A}{r}\right)^{-\beta_1^2 - 1} \exp \left\{-2\beta_1^2 (1-\beta_1^2) \log \left(\frac{A}{r}\right)^{-1} \right\} \right] \tag{4.31b}$$

$$p = k_1 \left[\log \frac{A}{r} + \gamma \overline{K}_4 \left(\log \frac{A}{r} \right)^{1-q_2} \right] \tag{4.32a}$$

$$p = k_1 \left[\log \frac{A}{r} + \gamma \overline{K}_4 \left(\log \frac{A}{r} \right)^{\beta_1^2} \right] \exp \left\{ - 2\beta_1^2 (1 - \beta_1^2) \log\left(\frac{A}{r}\right)^{-1} \right\} \Big] \tag{4.32b}$$

STRONG SHOCK WITH STRONG MAGNETIC FIELD

$$U = \frac{(N-1)}{N} \left[\overline{K}_5 r^{-\beta} + \frac{C}{B} \right]^{1/2} \tag{4.33a}$$

$$U = \frac{(N-1)}{N} \left[K_5 r^{-\beta} + \frac{C}{B} \right]^{1/2} \qquad (4.33b)$$

$\rho = \rho' N$ for converging diverging cases (4.34)

$$p = K_1 \log \frac{A}{r} \left[\frac{X(\overline{K_5} r^{-B} + C/B)}{K_2 (\log \frac{A}{r})^{1/2}} + L \right] \qquad (4.35a)$$

$$p = K_1 \log \frac{A}{r} \left[\frac{X(K_5 r^{-B} + C/B)}{K_2 (\log \frac{A}{r})^{1/2}} + L \right] \qquad (4.35b)$$

where section (a) and (b) denote the expression for converging and diverging shocks respectively.

REFERENCES

1. Kumar. S. & Prakash. R. Proc. Indian Soc. Theor. (1982)
   Appl. Mech 27(2), 11.

2. Kumar. S. & Prakash R. IL Nuovo Ciemento, Vol 77, N, 2 (1903)
   191-201.

3. Kumar. S. Saxena, A. K. & Prakash. R. Nuovo Ciemento (1986)
   7D

4. Chester. W. :
   Phil mag 45 (7) 1293 (1954)

5. Chisnell. R. F. :
   Proc. Roy, Soc, London Ser A232, 350 (1955)

6. Whitnam G. B. :
   J Fluid Mech. 4.337, (1958)

7. Singh J. B. & Pandey. K. S.
   Astrophysics and space science 148, 85 – 93, (1988)

CHAPTER - 5

ANALYSIS OF MAGNETO HYDRO DYNAMICS SPHERICALLY SHOCK WAVE IN A MEDIUM WITH VARIABLE DENSITY

INTRODUCTION

Pai [1], Green Span [2] Chester and Helliwell [3] have studied a cylindrical shock wave of line explosion, produced on account of a sudden release of a finite amount of energy expanding outwards in conducting gas subject to magnetic field. Deb Ray [5] and Singh & Vishwakarma [6] have obtained a similarity solution for strong cylindrical shock wave in a conducting non uniform medium Singh & Srivastava [6], have obtained the similarity solution for a axially symmetric explosion model in magneto gas dynamics I & II without thermal radiation. Singh & Pandey [7] discussed the conduction for the total energy of the cylindrical shock wave to be assumed as constant.

The present chapter deals with the problem in which the amount of energy is not finite in an infinite concentrated form is suddenly released with spherical symmetry in a conducting gas subjected to an azimuthal magnetic field which produce a spherical shock wave. This propagates in the medium whose density varies with time. The motion of the gas is assumed to be isothermal and total energy of the wave.

EQUATION OF MOTION AND BOUNDARY CONDITION

The equation of motion for one dimensional unsteady isothermal flow of a perfect gas with transverse magnetic field are

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + 2\rho\frac{u}{r} = 0 \qquad (5.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{h}{\rho}\frac{\partial h}{\partial r} + \frac{h^2}{\rho r} = 0 \qquad (5.2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + h\frac{u}{r} = 0 \qquad (5.3)$$

$$\frac{\partial T}{\partial r} = 0 \qquad (5.4)$$

The magnetic permeability is taken to be unity other symbol have their usual meanings and they are function of r and t only. Where V is ratio of specific heat.

Following Whitham(1959), we find the shock boundary conditions to be

$$u_1 = (1 - \frac{1}{N}) \qquad (5.5)$$

$$\rho_1 = N\rho_o \qquad (5.6)$$

$$H_1 = Nh_o \qquad (5.7)$$

$$P = \rho_1/\rho_o p_o \qquad (5.8)$$

$$N^2 + (2d + 1)N - 2M_A^2 = 0$$

$$D = \frac{1}{\gamma}\left(\frac{M_A}{M}\right)^2$$

Where N is dimensionless ratio, M_A is Alfen's Mach number and M_B is Mach number.

SIMILARITY SOLUTION

Let the similarity variable η be in the form

$$\eta = \frac{r}{R}, \quad R = R\,(t) \tag{5.9}$$

We seek the solution of the partial differential equation (1 – u) in the form

$$u = RV\,(\eta)\,,$$

$$\rho = \rho_o\, g\,(\eta)$$

$$p = \rho_o\, R^2\ f(\eta)$$

$$h = \rho^{1/2}\, R\, H\,(\eta) \tag{5.10}$$

where v, g, f and H are function of η the scale R, ρ_o and R time dependent in some manner as yet unknown let us assumed the radius and density of the cylindrical shock front are

$$R = A\, t^{\alpha} \tag{5.11}$$

$$\rho_o = \beta\, t^{\beta} \tag{5.12}$$

we now substitute the relation (5.10) in to continuous (5.1 – 5.4) taking account of the definition of similarity variable (5.9) and applying relations of the type.

$$\frac{\partial \rho}{\partial t} = g\rho_{o'} - \rho_o\, g'\eta\ \frac{R'}{R} \tag{5.13}$$

$$\frac{\partial h}{\partial t} = \frac{H}{2\rho_o}(2\ddot{R}\rho_o + \dot{R}\rho_o) - \rho_o^{1/2} R' \frac{2}{R\eta H'} \qquad (5.14)$$

$$\frac{\partial p}{\partial r} = \frac{\rho_o g'}{R} \qquad (5.15)$$

$$\frac{\partial h}{\partial H} = \rho_o^{1/2} \frac{R'}{RH'} \qquad (5.16)$$

to transform the derivatives the dot represent the differentiation of scales with respect to time and prime denote differentiation of functions with respect to similarity variable therefore the non differential form of ordinary differential equation and are

$$\frac{\beta}{\alpha} + \frac{v' + (v - \eta)g'}{g} + \frac{2v}{\eta} = 0 \qquad (5.17)$$

$$\frac{2(\alpha - 1) + \beta}{2\alpha} + \frac{(v - \eta)H'}{H} + V' = 0 \qquad (5.18)$$

$$\left(\frac{\alpha - 1}{\alpha}\right).v + (v - \eta)V' + \frac{1}{g}\left\{f' + H\left(H' + \frac{H}{\eta}\right)\right\} \qquad (5.19)$$

$$\frac{f'}{f} - \frac{g'}{g} = 0 \qquad (5.20)$$

consider now the total energy E of the flow as

$$E = 2\pi \int_{0}^{R} \left[\frac{1}{2\rho v^2} + \frac{p}{\gamma - 1} + \frac{h'^2}{2} \right] r.dr \qquad (5.21)$$

Equation (5.10) reduce the above equation in the form.

$$E = 2\,\pi\,\beta\,A^4\,\alpha^2\,t^{\,4\alpha + \beta - 2} \int_{\eta_o}^{1} \left(\frac{1}{2gv^2} + \frac{f}{\gamma - 1} + \frac{H2}{2} \right) \eta \qquad (5.22)$$

The total energy of the wave is constant so we have

$$\beta = 2 - 4\,\alpha \qquad (5.23)$$

it is clear from (5.23) that uniform expansion of wave take place, when $\beta = 2$ the solution have their physical significance only
when

$-2 < = \beta < = 0$

after using similarity transformation (5.10) in the jump condition (5.5 – 5.8) at the shock from yield

$$g\,(1) = f\,(1) = N$$

$$H\,(1) = \frac{N}{M_A} \qquad (5.24)$$

$$V\,(1) = (1 - 1/N)$$

Using R.K.G's method starting from shock front with transformed boundary condition and counting up to time explosion equation are integrated. We have calculated our result in the form.

$$\frac{U}{U_1} = \frac{v(\eta)}{(1 - 1/N)}$$

$$\frac{P}{p_1} = \frac{\rho}{\rho_1} = \frac{g(\eta)}{(1-1/N)} \qquad (5.25)$$

$$\frac{h}{h_1} = \frac{M_A \cdot H(\eta)}{N}$$

ADIABATIC CASE

The equation of motion and field for perfect gas with unsteady adiabatic flow with transverse magnetic field are in addition to equation (5.1) to (5.3) and

$$\frac{\partial}{\partial t} = (l_n P \rho^{-\gamma}) + u \frac{\partial}{\partial r} (l_n P \rho^{-\gamma}) = 0 \qquad (5.26)$$

as usual the magnetic permeability of the medium is taken to be unity following with an (5.1) the shock boundary condition are taken to be

$$\rho_1 = p\, \rho_0$$

$$u_1 = \psi h$$

$$h_1 = \psi h$$

$$p_1 = \left[\frac{1}{\gamma M^2} + \frac{2(\psi - \gamma)}{(\gamma + 1)(\gamma - 1)} \psi \left\{ \frac{1}{M^2} + \frac{(\gamma - 1)(\psi - 1)^2}{4 M_A^2} \right\} \right] \rho_0 R'^2 \qquad (5.27)$$

where ψ can be calculated from quardratic equation

$$\left[\frac{2 - \gamma}{M_A^2} \right] \psi + \left[\frac{2}{m^2} + \frac{\gamma}{M_A^2} + (\gamma + 1) \right] \psi + (\gamma + 1) = 0 \qquad (5.28)$$

and

$$M_A^2 = \frac{\rho R'^2}{h^2}, \qquad M^2 = \frac{\rho_0 R'^2}{\gamma p_0} \qquad (5.29)$$

SIMILARITY SOLUTION

It is convenient to assume shock radius and density varying as time

$$R = C e^{mt}, \quad \rho = D e^{nt} \qquad (5.30)$$

Where m, n are constants.

Let the similarity variable η be in the form

$$\eta = \frac{r}{R}, \quad R = R(t)$$

Or

$$\eta = \frac{r\, e^{-mt}}{c} \qquad (5.31)$$

We seek this solution of partial different equations (5.1 to 5.3) and (5.26) in the form given in (5.10)

$\mu = R'\, v\, (\eta)$

$\rho = \rho_o\, g\, (\eta)$

$p = \rho_o\, R'2\, f\, (\, \eta\,)$

$h = \rho_o ½\, R'\, H\, (\, \eta\,)$

where v, g, f and H are function of η

We now substitute relation (5.10) in the equation (5.1 – 5.3) and (5.26) and taking up to account of similarity variable (5.31) and applying the relation (5.31 – 5.16) we find the dimensional form of ordinary differential equation which yield by considering the possibility is

$$R\, R'' / R'^2 \quad = \quad \text{Constant}$$

and

$$\rho' / \rho_o \quad = \quad \text{Constant}$$

The time dependent equations (on account of parameter η) are

$$\frac{N}{m} + \frac{(v-\eta)g'}{g} + v' + \frac{v}{\eta} = 0 \qquad (5.32)$$

$$v + v'(v - \eta) = \frac{1}{g}\left(f' + H H' + \frac{H'}{\eta}\right) = 0 \qquad (5.33)$$

$$\left(\frac{\eta}{2m} + 1\right) + \frac{(v - \eta) H'}{H} + v' = 0 \qquad (5.34)$$

$$2 f \frac{n}{m} + (v - \eta)\left(\frac{f'}{f} - \frac{vg'}{g}\right) = 0 \qquad (5.35)$$

where dash denotes differentiation with respect to "η" considering the total energy E of the flow as

$$E = 2\pi \int_{o}^{R} \left(\frac{1}{2}\rho u^2 + \frac{\rho}{r} - 1 + \frac{h^2}{2}\right) r.dr \qquad (5.36)$$

Equation 5.30, 5.31 and 5.10 reduce the above equation in the form

$$E = 2\pi D' C^4 e^{(n^{-1} 4m)t} \int_{\eta_o}^{1} \left(\frac{gv^2}{2} + \frac{f}{\gamma-1} + \frac{H^2}{2}\right) \eta d_{\eta} \qquad (5.37)$$

In order to have total energy of the wave is constant. We have

$$\eta = -4m \qquad (5.38)$$

It is observe that uniform expansion of wave take place

When $\frac{n}{m} = -4$

The solution their physical significance only when

$$-4 <= n <= 0 \tag{5.39}$$

After using similarity transformation jump conditions at shock proof yield.

$$g(1) = \psi$$

$$v(1) = (1 - 1/\psi) \tag{5.40}$$

$$f(1) = k\psi$$

$$H(1) = \psi/M_A$$

Using R.K.G. Method from shock front with transformed boundary conditions and containing up to line of explosion for a adiabatic case equation (5.32) – (5.35) are integrated numerically on R. K. G's program in the

$$\frac{V}{u_1} = \frac{v(\eta)}{(1-1/\psi)}$$

$$\frac{\rho}{\rho_1} = \frac{g(\eta)}{\rho}$$

$$\frac{p}{p_1} = \frac{f(\eta)}{K} \tag{5.41}$$

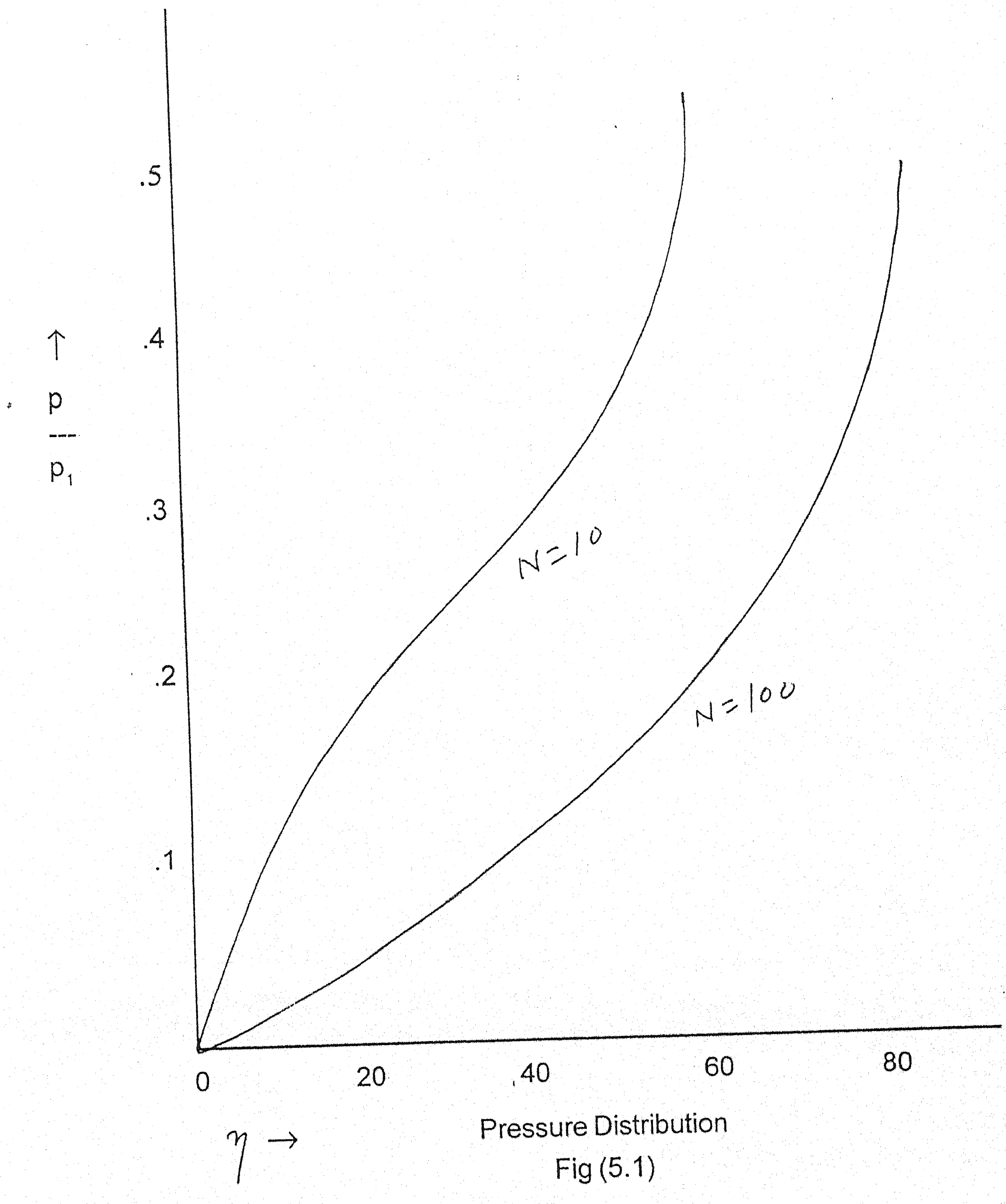
$$\frac{h}{h_1} = \frac{M_A H(\eta)}{\psi}$$

RESULT AND DISCUSSION

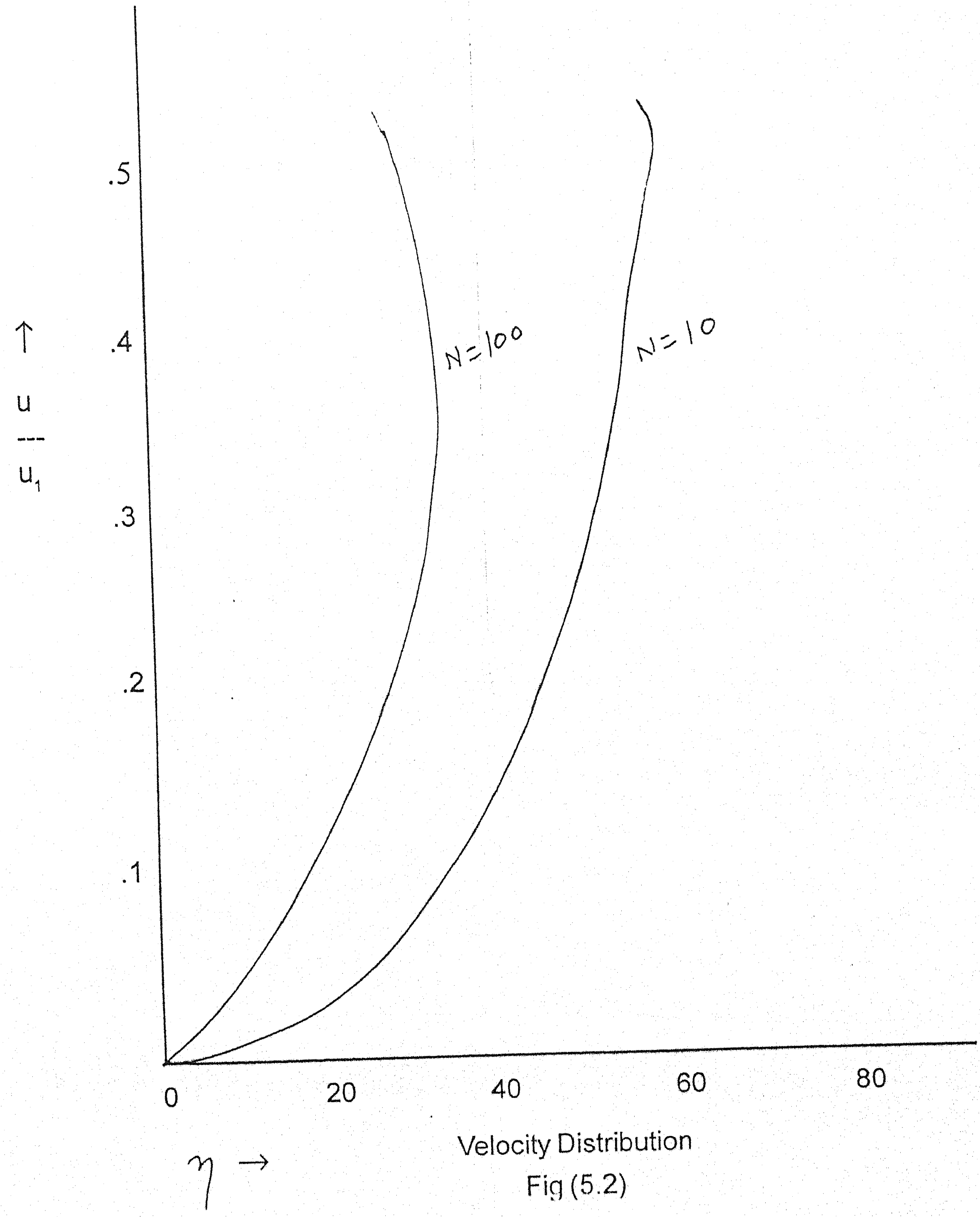
The Equation of motion from (5.1 – 5.4) under the given boundary condition (5.5 – 5.8) are integrated with the help of (5.13 – 5.16) which under the similarity condition given equation number (5.10) which yield to non dimensional ordinary differential equation (5.7 – 5.20) for the propagation of Magneto Hydro Dynamics spherical shock wave. In an unsteady isothermal flow behind the perfect gas.

The numerical values are plotted against the graph by fig (5·1) to fig (5·4) for the various values of N = 10 , 100 the graph clearly indicates that the pressure, velocity, density distribution varies exponentially as we move away from the shock surface where as magnetic distribution tends for N = 10 it limits indefinite range 0.1 to 0.02 again it exponentially for the large value of N = 100.

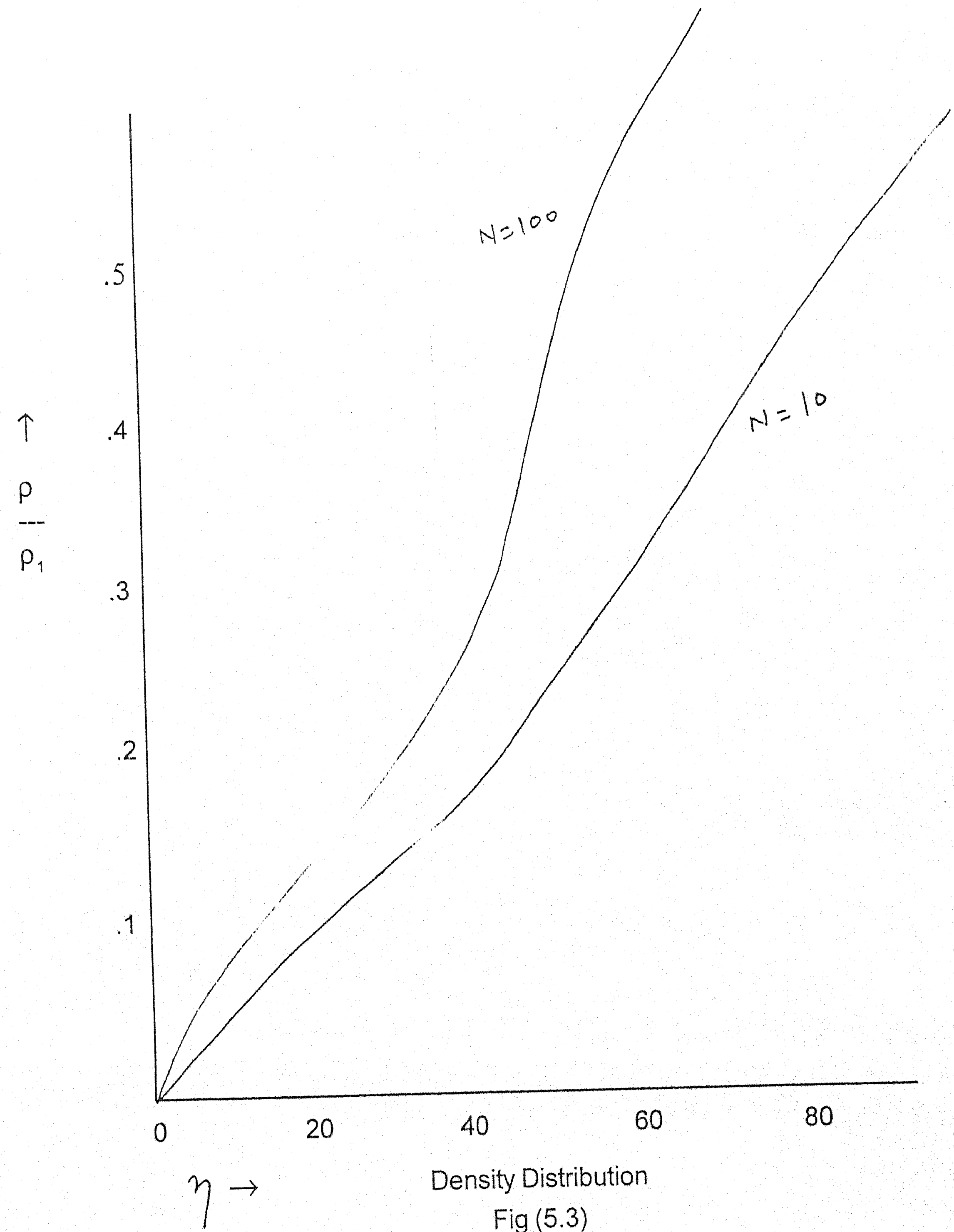
When flow is isothermal



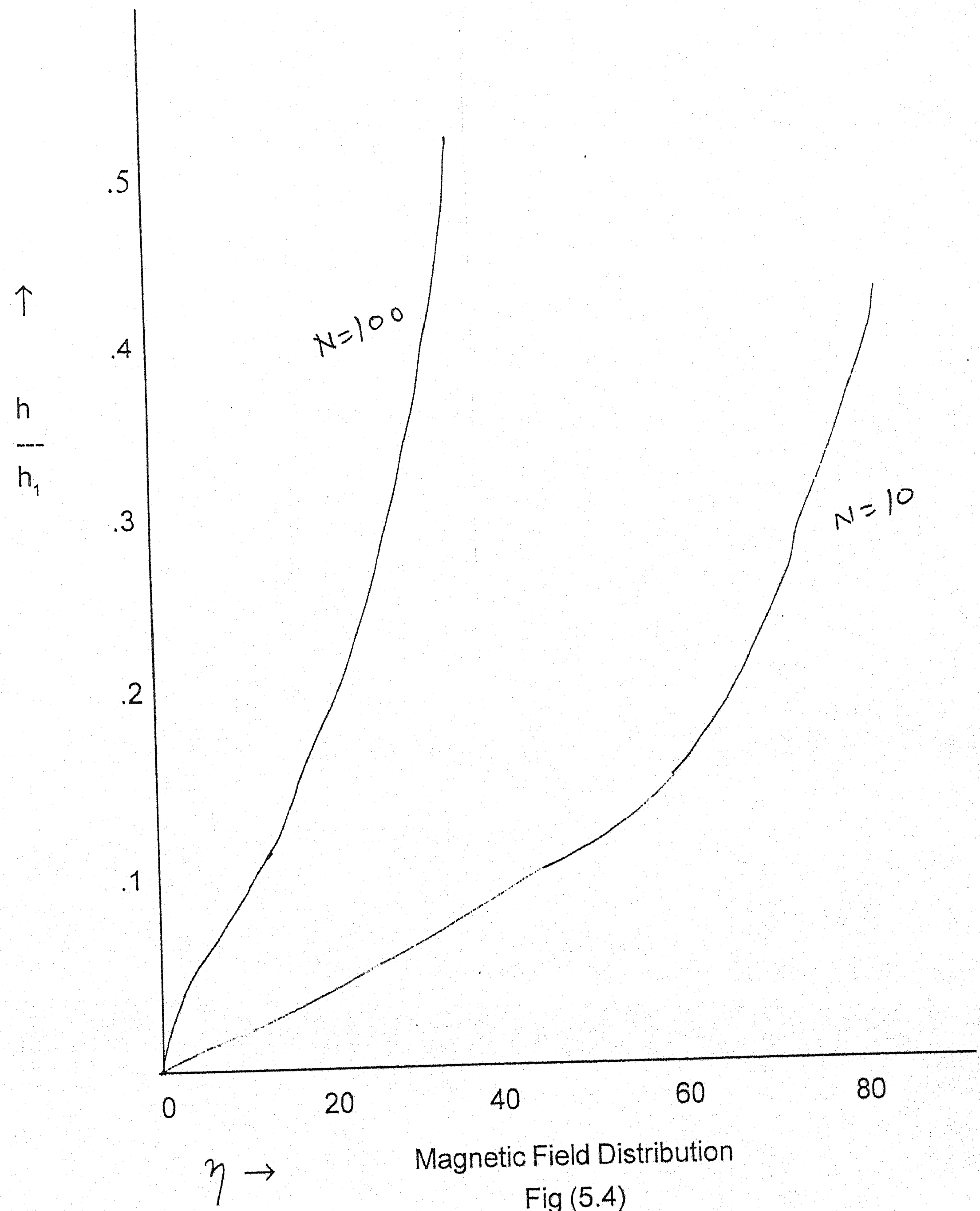
Pressure Distribution
Fig (5.1)



Velocity Distribution
Fig (5.2)



Density Distribution
Fig (5.3)



Magnetic Field Distribution
Fig (5.4)

REFERENCES

1. Pai SI : Proc Forth Cong. Theory. Appl. Mech (89) 134 (1958)

2. Greenspan H.P. : Phys fluids (5) 255, (1962)

3. Christer A. H. & Holliwell J. B. : J Fluid Mech. (39) 705(1969)

4. Helliwell J. B. J Fluid Mech (37) 497 (1969)

5. Deb Ray G. : Physics Fluid (16) 559 (1973)

6. Singh J. B. & Vishwakarma P.R. : Astrophysics and Space Science (93) 423 (1983)

7. Singh J. B. & Pandey S. K. : Astrophysics and Space Science (146) 71-78 (1988)

CHAPTER – 6

PROPAGATION OF PLANE SHOCK WAVE IN MAGNETO GAS DYNAMICS

INTRODUCTION

The phenomena associated with heat transfer in rotating fluids are extremely complex whether the fluid is at rest or in motion, study or unstudy Wang [9] considered the piston problem with thermal radiation for one dimensional unstudy shocks using the similar method of Sedov [6], Hellwel [2] took a more general case of piston problem with radiation heat transfer for general optically and transparent limit. Elliot [1] discussed the self similar solution for spherical blast waves in air using the Rossland diffusion approximation under the assumption that there is no effect of heat flux at the centre of symmetry laumnch and Probrteim [4] have studied the radiation effects on shock waves in an spherical medium several other author e.g. summer [8] magnetic field also be taken into account. The unsteady model of Roche – Cosits of a gas distributed with spherical symmetry around a nucleous of large mass. It is assumed that the gravitating effect of the gas neglected compared with the attraction of the heavy nucleus Rosenau [5].

In this chapter we consider propagation of plane shock wave under isothermal condition. In the isothermal condition temperature gradient becomes zero behind the shock and radiation effect are already implicity involved. An idealized magnetic field is considered for only a portion of sphere including the origin i.e. the point of explosion Summer [8] the field in directly tangential to the advancing shock front. The shock is assumed to advance into conducting gas of specially decreasing density and produced by a specially decreasing magnetic field the total energy of the above in non – constant.

EQUATION OF MOTION FOR ADIABATIC FLOW

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} = 0 \qquad (6.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho} \frac{\partial h}{\partial r} + \frac{h^2}{\partial r} + \frac{Gm}{r^2} = 0 \qquad (6.2)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + \gamma p \frac{\partial p}{\partial r} + (\gamma - 1) \frac{\partial f}{\partial r} = 0 \qquad (6.3)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + h \frac{u}{r} = 0 \qquad (6.4)$$

where u, p, ρ, h and f are the velocity, pressure, density, magnetic field and radiative heat flux at a radial distance r from the center of the core at time t, m, denotes the constant mass of the core, G be the gravitational constant, the magnetic permeability of the medium has taken to emity.

The equation of state for ideal gas is given by

$$P = \Gamma \rho T \qquad (6.5)$$

Where Γ is the gas constant.

Also assuming local thermodynamics equilibrium and taken Rossland's diffusion approximation, we have

$$F = \frac{-C\mu}{3} \frac{\partial}{\partial r} (aT^4) \qquad (6.6)$$

Where (1/4) ac is the Stefan – Boltz man constant, C is the velocity of light and μ the mean free path of radiation, is a function of density and absolute temperature T, following Wang K. C. (9)

$$\mu = \mu_0 \rho^{\alpha} T^{\beta} \qquad (6.7)$$

where μ_0, α, and β being constant, the total energy of the wave is dependent on time as

$$E = E_o t^q \qquad (0 \le q \le 4/3) \qquad (6.8)$$

Where E_o being the constant in the front of shock wave, we assume that

$$\rho_1 = AR^{-\omega} \qquad (0 \le \omega \le 2) \qquad (6.9)$$

where A is constant and R denotes the radius of the shock surface ahead the shock, the magnetic field distribution is assumed to be as

$$h_1 = CR^{-k} \qquad (2K = \omega + 1) \qquad (6.10)$$

At the equilibrium state the other flow variable ahead the shock are

$$U_1 = 0, \qquad \rho_1 = \left[\frac{AmG}{1+\omega}\right] . R^{-(1+\omega)} + \frac{C^2}{K(1-k).\ R^{-2k}} \qquad (6.11)$$

where C, W and K are constant the disturbance is heated by an isothermal shock and Ranking Hugoniot jump condition are.

$$\rho_2 (v - u_2) = \rho_1 v = m_s \qquad (6.12)$$

$$p_2 + \frac{h_2^2}{2 - p_1} - \frac{h_2^2}{2} = M_s \, . \, u_2 \qquad (6.13)$$

$$h_2 (v - u_2) = h_1 v \qquad (6.14)$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (v - u_2)^2 \quad \frac{h_2^2}{2} - \frac{F_2}{M_s} = e_1 + \frac{p_1}{\rho_1} + \frac{v_2}{2} + \frac{h_1^2}{\rho_1} \qquad (6.15)$$

$$T_2 = T_1 \qquad (6.16)$$

Where suffix 2 denote the flow variable just behind the shock denotes the mass flux per unit, across the shock and v be the velocity of the shock and is given by

$$V = \frac{dR}{dt} \qquad (6.17)$$

SOLUTION OF THE EQUATION OF MOTION

In order to reduce the equation of flow to ordinary differential equation, we now introduce the following similarity transformation Sedov [6]

$$\eta = (VmG)^{-1/2} \, rt^{-\delta} \qquad (6.18)$$

where

$$\delta = \frac{2}{3} = \frac{(2+q)}{(5-\omega)}$$

i.e

$$q = \frac{2}{3}(2-\omega) \qquad (6.19)$$

We see the solution of equation (6.4 – 6.19) in the form

$$U = \frac{r}{t}\, v(\eta)$$

$$\rho = \frac{(Am\, Gt^2)}{r^{\omega+3}}\, D(\eta)$$

$$p = \frac{(Am\, G)}{r^{\omega+3}}\, p(\eta)$$

$$h = \sqrt{\frac{(Am\, G)}{r^{(\omega+1)2}}}\, N(\eta)$$

$$f = \frac{(Am\, G)}{r^{\omega}}\, Q(\eta) \qquad (6.20)$$

Where V is constant to be determined by the condition that assume the value 1 the shock front hence we have

$$R = (\,v\,m\,G)^{1/3}\,t^{2/3} \tag{6.21}$$

By use of equation (6.18) in equation (6.6) with the add of equation (6.5) we obtain

$$\alpha = \frac{\omega}{\omega + 1} \quad \& \quad \beta = \frac{(5\omega + 7)}{2\,(\omega + 1)} \tag{6.22}$$

Equation (6.1 – 6.4) and equation 6.6 are then transformed with the relation (6.18) and (6.20) to the form.

$$\frac{du}{d\eta} = p\,\frac{[Q\,.\,(v - \delta)]}{(D^{\alpha-\beta-4}\,\rho^{\beta+4})} - \omega\,v + \frac{2\{v\delta + 1\} - (\omega + 1)\,(v - \delta) + (v + \delta)}{\eta^2\,v}\ X$$

$$\frac{\eta[\,(v - \delta)^2\,D - P - N^2 + N^2\,[\,v\,(3 - \omega) - (v - \delta)\,(1 - \omega) + v\,(\,v - 1\,)\,(\,v - \delta\,)\,D}{\eta\,[\,v + \delta)^2\,D - P - N^2\,D}$$

(6.23)

$$\frac{dD}{d\eta} = \frac{D}{(v-\delta)}\ \frac{[\,(\omega\,v - 2)]}{\eta} - \frac{dv}{d\eta} \tag{6.24}$$

$$\frac{dp}{d\eta} = p\,\frac{1}{\eta}\,\left\{\,\frac{(\omega v - 2)}{(\,v - \delta)} - \frac{Q}{LD^{\alpha-\beta-4}\,.\,p^{\beta+4} - 2}\right\} - \frac{1}{(\,v - \partial)}\,.\,\frac{dv}{d\eta} \tag{6.25}$$

$$\frac{dN}{d\eta} = -\frac{N}{(v-\delta)}\left[\frac{v(3-\omega)}{2\eta} + \frac{dv}{d\eta}\right] \qquad (6.26)$$

$$\frac{dQ}{d\eta} = \frac{\gamma\rho}{(\gamma-1)\eta}\left[\frac{(v-\delta)(2+Q)}{LD^{\alpha-\beta-4}p^{\beta+4}} - \frac{v}{\gamma}\{\omega(\gamma-1)+3\gamma-1\}+2\right]$$

$$-\frac{Q}{\eta}(2-\omega) + \frac{dp}{d\eta}(\gamma-\delta) \qquad (6.27)$$

$$\text{where } L = \left(\frac{4\,c\,a\,\mu_o}{3\Gamma^{\beta+4}}\right)(AGM)^{\alpha-1} \qquad (6.28)$$

is a dimensionless radiation parameter.

EQUATION OF MOTION FOR ISOTHERMAL FLOW

The equation of continuity, momentum and magnetic field are the same as in the adiabatic flow but the energy equation becomes

T = Constant

$$\frac{\partial T}{\partial r} = 0 \qquad (6.29)$$

By use of equation[6.5] we have

$$\frac{P}{P_2} = \frac{\rho}{\rho_2} \tag{6.30}$$

By substitution of relation (6.18), (6.20) and (6.22) in equation (6.1 – 6.3) as (6.28) the following set of equation are obtained

$$\frac{dv}{d\eta} = p\,\frac{[\,2\,(1 - v - \delta) +'(v - \delta)\ (1 + \omega) - v] + N^2 X\ \ [\,v\,(3 - \omega) - (1 + \omega)\ (v - \delta)\,]}{\{\,\eta\,[\,v - \delta)^2\,D - N^2 - P)\,\}}$$

$$+ \frac{(v\ - \delta)\,D\,[\,v\,(\,v - 1) + \frac{1}{\eta^2 v}\,]\,\}}{\{\eta\,(\,v - \delta)^2\,D - N^2 - P\,]\,\}} \tag{6.31}$$

$$\frac{dD}{d\eta} = \frac{D}{(\,v - \delta\,)}[\,\frac{1}{\eta\,(\omega\,v - 2)} - \frac{dv}{d\eta}\,] \tag{6.32}$$

$$\frac{d\rho}{d\eta} = p\,[\,\frac{1}{D})\ \ \frac{dD}{d\eta} - \frac{2}{\eta}\ \ ] \tag{6.33}$$

$$\frac{dN}{d\eta} = -\,\frac{N}{(\,v - \delta)}\,[\,(\frac{v}{\eta}\,)\,\frac{(3 - \omega)}{2} + \frac{dv}{d\eta} \tag{6.34}$$

For the isothermal flow we also assume that the jump condition are heated by isothermal flow as given by equation 6.12 -- 6.16.

RESULT AND DISCUSSION

The transformed jump condition at the shock for both adiabatic and isothermal flow are given by

$$V(1) = \left(\frac{2}{3}\right)\Omega \tag{6.35}$$

$$D(1) = \frac{v}{(1-\Omega)} \tag{6.36}$$

$$P(1) = \frac{4v}{9}\left[\frac{\Omega + 1}{\gamma M^2} + \frac{\Omega(2-\Omega)M^2_A}{\{2(1-\Omega)^2\}}\right] \tag{6.37}$$

$$N(1) = \frac{2}{3}\sqrt{v}\left(\frac{1}{1-\Omega}\right)M'_A \tag{6.38}$$

and

$$Q(1) = \frac{8v}{37}\Omega\left\{\frac{(1-\Omega)\gamma M^2 - 1}{(\gamma - 1)M^2}\right\} + \left\{\frac{M^2_A(\gamma\Omega - 2)\Omega}{2(1-\Omega)(\gamma-1)} + \frac{(2-\Omega)\Omega}{2}\right\} \tag{6.39}$$

Where V is obtained from the relation

$$V = \left(\frac{9}{2}\right) v M^2 \left[2(1+\Omega) - \gamma\left(\frac{M^2}{M_A}\right)(1-\Omega)\right]^{-1} \tag{6.40}$$

For exhibiting the numerical solution, it is convenient to write variable in non dimensional form as.

$$\frac{u}{u_2} = \eta \left[\frac{v(\eta)}{v(1)} \right] \tag{6.41}$$

$$\frac{\rho}{\rho_2} = \frac{1}{\eta^{\omega+3}} \left[\frac{D(\eta)}{D(1)} \right] \tag{6.42}$$

$$\frac{p}{p_2} = \frac{1}{\eta^{\omega+1}} \left[\frac{p(\eta)}{p(1)} \right] \tag{6.43}$$

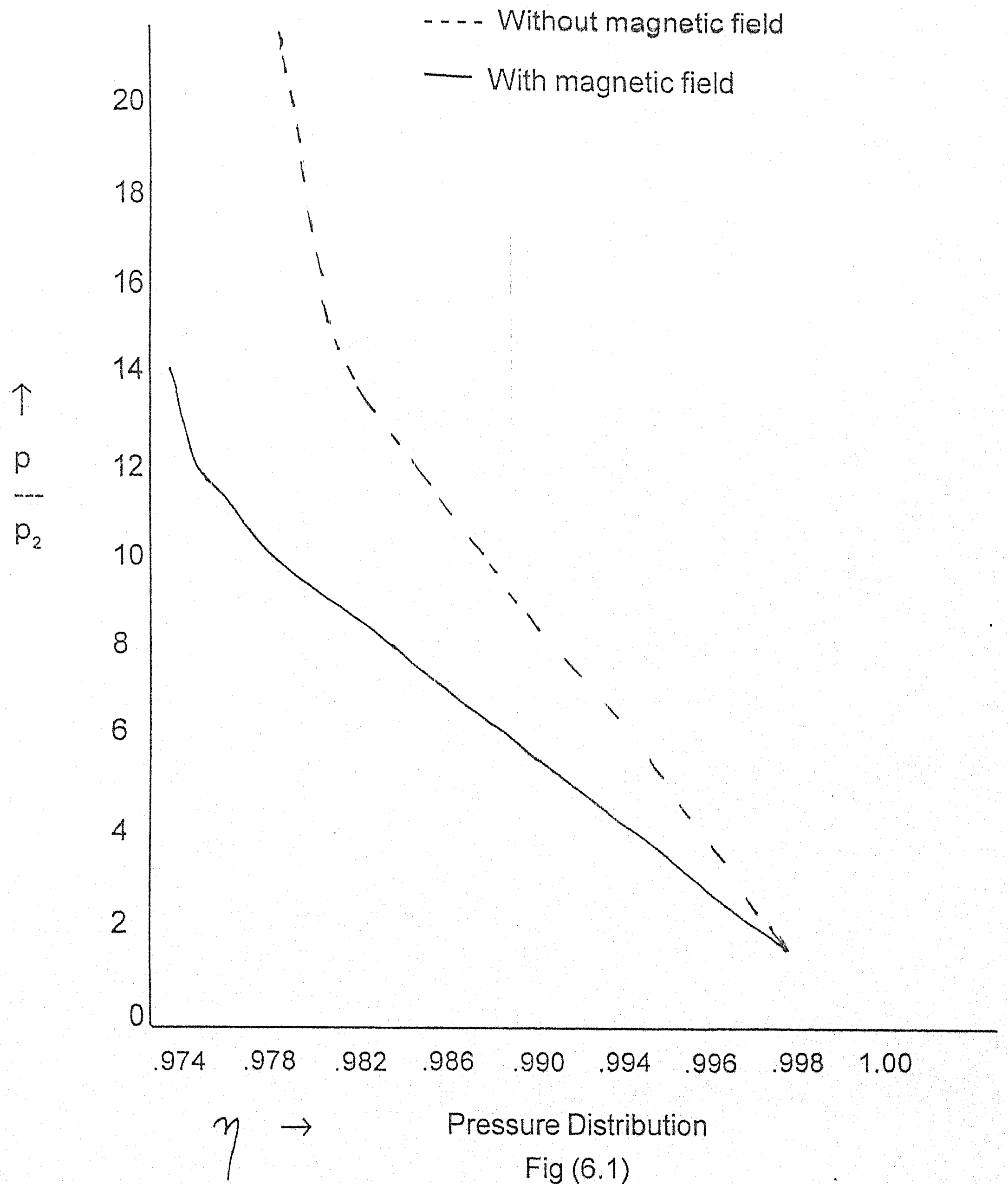
$$\frac{h}{h_2} = \frac{1}{\eta(1+\omega)^{2}} \left[\frac{N(\eta)}{N(1)} \right] \tag{6.44}$$

and

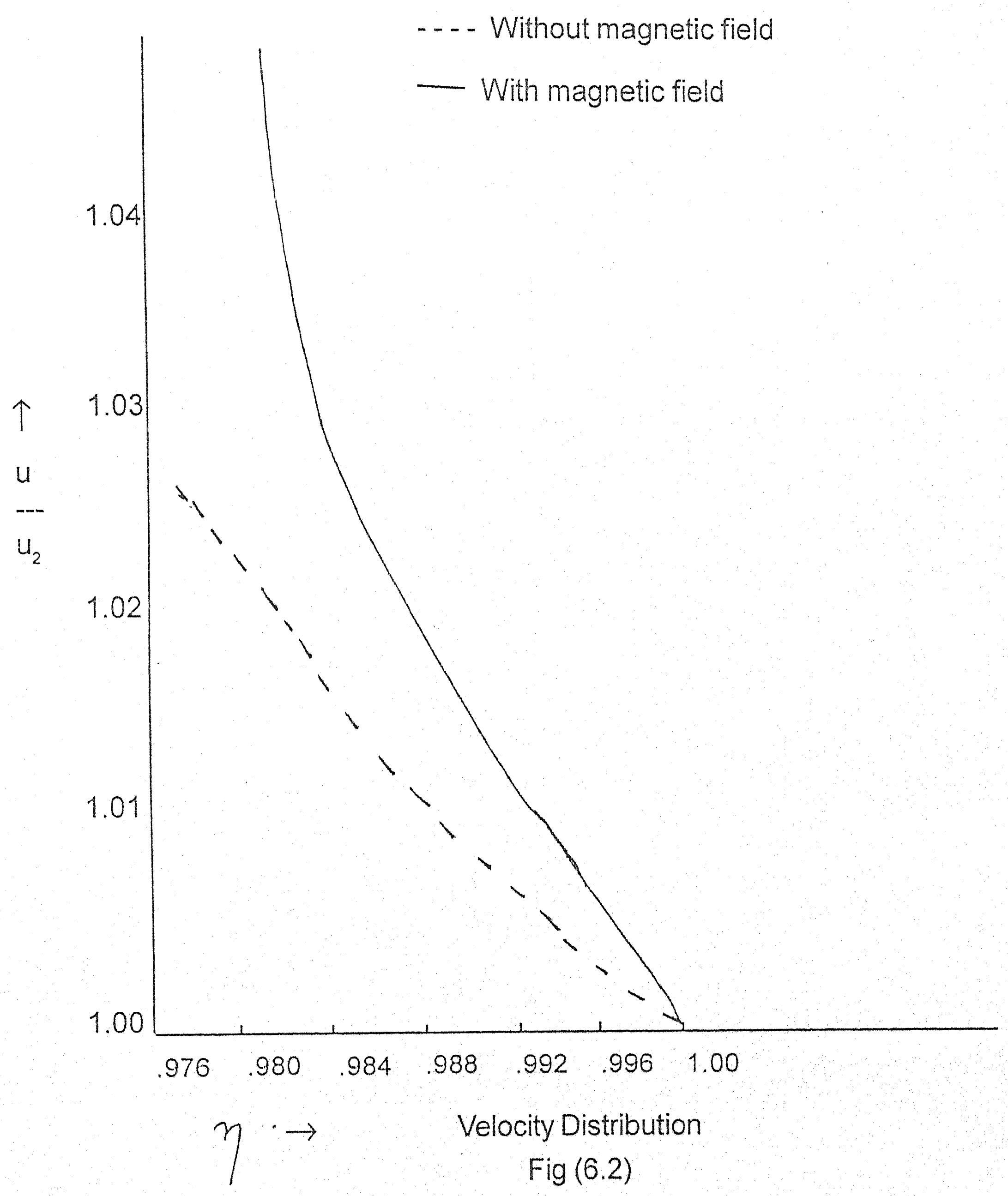
$$\frac{F}{F_2} = \frac{1}{\eta^{\omega}} \left[\frac{Q(\eta)}{Q(1)} \right] \tag{6.45}$$

The numerical integration is carried out for boath adiabatic and isothermal flow separately on DEC – system 1090 computer installed at IIT Kanpur by the well known RKG system programmer for the there values W = 1.00, W = 1.50 and W = 2.00. The other constants are $M^2 = 20$, $M^2_A = 10$, $\delta = 2/3$, and L = 10.

The nature of flow and field variable for both the adiabatic and isothermal cases are illustrated through figure[6.1 – 6.4] it is clear from the figure that velocity, density, pressure and magnetic field are maximum at the shock front and decreases rapidly towards the centre of explosion, when the flow behind the shock wave is isothermal but these variables are minimum at the shock front and increases steadly so we move towards the centre of explosion in adiabatic case. The condition of radiation heat flux has been illustrated in figure 5 we therefore conclude that the flow and field parameters are being subsidied, when the assumption of adiabatic to be not valid and the temperature gradient of the flow becomes zero.

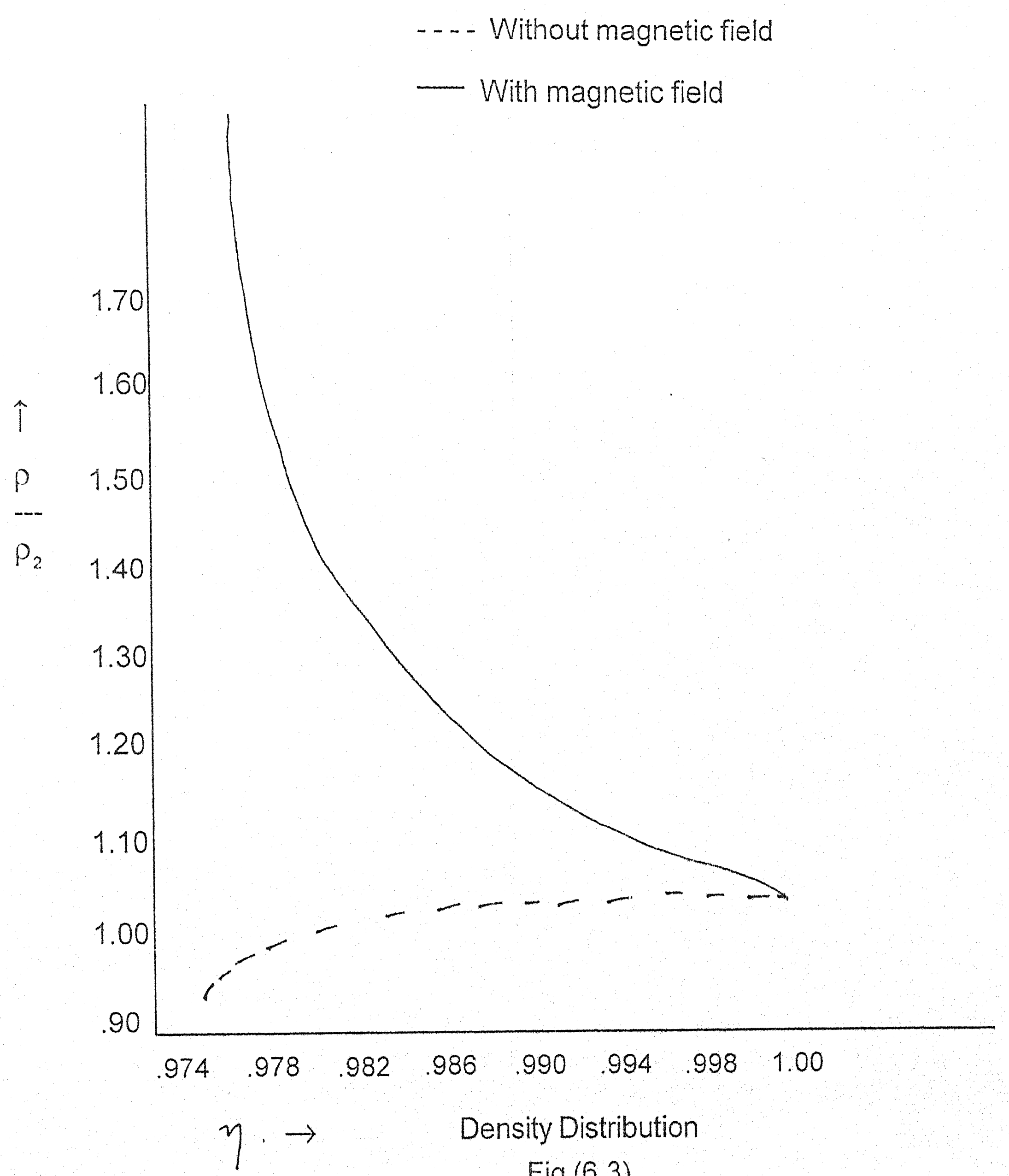


Pressure Distribution
Fig (6.1)



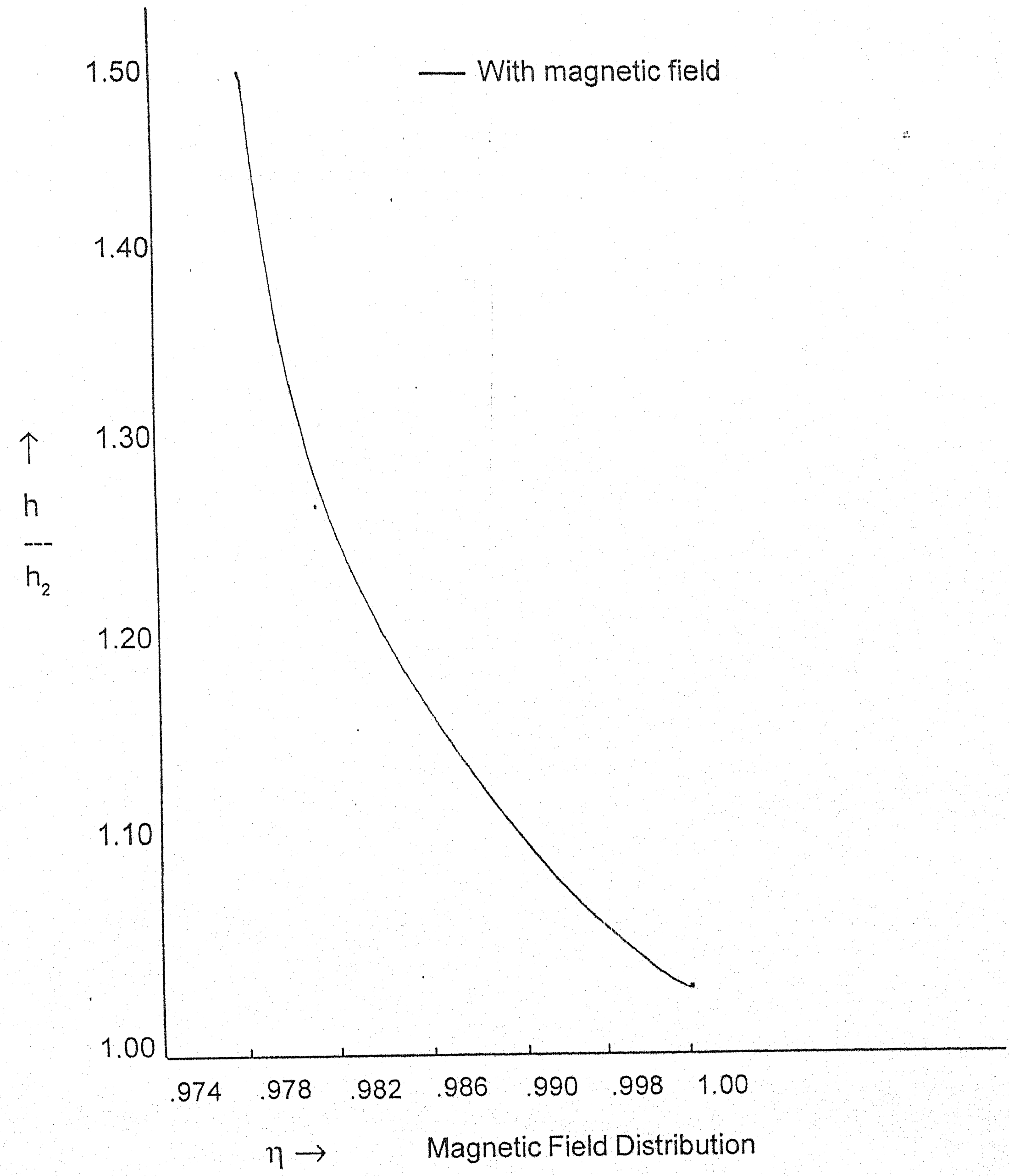
Velocity Distribution

Fig (6.2)



Density Distribution

Fig (6.3)



Magnetic Field Distribution

Fig (6.4)

REFERENCES

1.	Elliot L.A.	:	Proc. Roy. Soc. 258A, 287	(1960)
2.	Hellint J.B	:	Jn. Fluid Mech 37, 497	(1969)
3.	Koch P.A.	:	Physics Fluid 8, 2140	(1959)
4.	Laumbch D.D & Probrteim RF	:	Jn. Fluid Mech. 35, 53	(1969)
5.	Roshon P.	:	Physics Fluid 20, 1097	(1977)
6.	Sedov L.I	:	Similarity and dimensional method in mechanics, Academic Press New York	(1959)
7.	Singh J.B & Vishwakarma P.R.	:	Astrophysics and space science 90, 161	(1983)
8.	Summers. D.	:	Astrophysics and space science 40, 151	(1975)
9.	Wang K.C.	:	Jn. Fluid Mech 40, 447	(1964)

CHAPTER – 7

RADIATIVE EFFECT OF POINT EXPLOSION IN STELLAR BODIES

INTRODUCTION

Carrus etal [1] and Sedov [2] were first to discuse the bodies of stellar explosion in which a star considered to be a perfect self gravitating gas.

In this chapter we consider the some point explosion in stellar bodies since the temperature of material is very high the radiation effect can not be ignore. The aim in this chapter is to discuse the equation for the propagation of a radiative blast wave produced by a sudden point explosion in self gravitating system such as starts. We have taken similarity solution and medium has been assumed to be perfectly conducting plasma with radiative parameters independent of magnetic field. The disturbance is bounded on the outside. In this case also the viscosity and heat conduction are neglected and it is assumed that the flow is isentropic along a stream line, Lee [4] discussed the model of blast wave model to account for interior energy. In this chapter we also consider effect of transvers magnetic field in stellar bodies.

EQUATION OF MOTION

The equation of motion, continuity, energy and the filed equation in the case of a radiative gas are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{H_0}{\rho}\frac{\partial H_0}{r} + \frac{Gm}{r^2} = 0 \qquad (7.1)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (7.2)$$

$$\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial r} + \rho\left(\frac{\partial}{\partial t}(1/\rho)+u\frac{\partial}{\partial r}(1/\rho) + \frac{1}{\rho r^2}\frac{\partial(fr^2)}{\partial r}\right) = 0 \qquad (7.3)$$

$$\frac{\partial H_0}{\partial t} + u\frac{\partial H_0}{\partial r} + H_0\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \qquad (7.4)$$

where

$$E = \frac{H_0^2}{2\rho}, \qquad P = \frac{H_0^2}{2} \qquad (7.5)$$

The quantities u, p and ρ are radial velocity, pressure and density at a distance r at any time from the point of explosion the magnetic field has component (0, 0,H_0), f is the radiation flux. G is the gravitational constant and m is the mass within the shock front at any time t such that

$$\frac{\partial m}{\partial r} = 4\pi\rho\, r^2 \qquad (7.6)$$

We have

$$E = \frac{P}{\rho^{(\gamma-1)}} \qquad (7.7)$$

and

$$F = \frac{\varepsilon}{\rho}, \quad \frac{dp}{dr} \qquad (7.8)$$

where ε is the cofficient of opacity, C the velocity of light, we assume as in [5]

$$P = ZP, \qquad (0 < Z < 1)$$

So that

$$E = \frac{P}{\rho^{(k-1)}} \tag{7.9}$$

where K is called Klimishin's cofficient and is given by [5]

$$K = \frac{4(\gamma-1) + Z\,(4-3\gamma)}{3(\gamma-1) + Z\,(G-3\gamma)} \tag{7.10}$$

γ being the usual ratio of specific heats, with the help of (7.9) , (7.2) and (7.4) and (7.3) can be written as

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + K\rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) + \frac{k-1}{r^2}\,\frac{\partial(fr^2)}{\partial r} = 0 \tag{7.11}$$

let the motion be assumed to be confined within the shock front at r = R(t) then, the velocity of shock moving outwards is given by

$$V = \frac{dR}{dt} \tag{7.12}$$

SELF SIMILAR VARIABLES

The following similarity forms are used for flow and field variables.

$P =$	$P_0\ R^m\ f(\eta)$	(7.16)
$\rho =$	$\rho_0\ \psi\ (\eta)$	(7.14)
$u =$	$R^n\ \varphi\ (\eta)$	(7.15)
$H =$	$H_o\ R^k g\ (\eta)$	(7.16)
$F =$	$f_o\ R^l\ n\ (\eta)$	(7.17)
$m =$	$m_o\ R\ m\ (\eta)$	(7.18)

where $\eta = r/R$ is a nondimensional radial variable f, ψ, φ, g, m, n are function of η only and the suffix zero denotes quantities in the undisturbed state, putting the equation (7.1) – (7.4) and (7.6) into their similarity form, we get

$$R / R^{n} (\eta\varphi - \eta\varphi') + \varphi' \varphi = 1/\rho_0\psi$$

$$P_o R^{(m-2n)} F' + H_o R^{(2k-2n)} g' + m_o R^{(b-2n-1)} m(\rho_0 \psi) \tag{7.19}$$

$$\frac{\psi}{\eta}(n\varphi' + 2\varphi) = P\left(n\frac{R}{K^{n}} - \varphi\right) \tag{7.20}$$

$$\frac{R}{R^{n}}(mf' - \eta' f_2) = K\left(\varphi' f + \frac{2\varphi f}{\eta}\right) + \frac{(k-1)f_o}{\rho_o} R^{l-m-n}\left(\eta\,\eta' + \frac{2n}{\eta}\right) \tag{7.21}$$

$$\frac{R}{R^{n}}\left(\eta k - \eta^{2}\frac{g'}{g}\right) + \eta\varphi_1\frac{g'}{g} + \eta\varphi' + 2\varphi = 0 \tag{7.22}$$

$$m' = \frac{4\pi\rho_o}{m_o} R^{-b+3}\psi\eta^{2} \tag{7.23}$$

in order that all the unknowns may be expressible as function of η along the following relation must be fulfilled.

$$m = 2n, \qquad k=n, \qquad L=3n, \qquad b=2n+1 \tag{7.24}$$

and

$$\frac{R}{R^{n}} = C \tag{7.25}$$

Integrating (7.25) we get

$$ct = \frac{R^{(1-n)}}{(1-n)} + A$$

n being an arbatrary parameter and C an absolute constant, As t→0 and R→0 and so we must have $n<1$ and $A=0$, then shock radius R is given as

$$R = [(n-1)(ct)]^{1/1-n} \qquad n<1 \qquad (7.26)$$

with the help (7.24) (7.25) and (7.26) the equation (7.19) – (7.23) become

$$C(\eta\varphi - \eta\varphi') + \varphi'\varphi = -1/\rho_o\psi$$

$$\rho_o f' + H_o g' + m_o m' \rho_o \psi \qquad (7.27)$$

$$\frac{\psi}{\eta}(\eta\varphi' + 2\varphi') = \varphi'(\eta c - \varphi) \qquad (7.28)$$

$$C(2\eta f - \eta f') = K\left(f\varphi' + \frac{2\varphi f}{\eta}\right) + \frac{(k-1)f_o}{p_o}\left(n'\eta + \frac{2n}{\eta}\right) \qquad (7.29)$$

$$C\left(n\eta - \frac{g'\eta^2}{g}\right) + \eta\frac{m\,g'}{g} + \eta\varphi' + 2\varphi = 0 \qquad (7.30)$$

and the similarity transformation (7.13) to (7.18) becomes

$$\left.\begin{aligned}
p &= p_o R^{2n} f(\eta),\\
u &= R^n R\ (\eta),\\
F &= R_o R^{2n}\ n\ (\eta)\\
m &= m_o R^{(2n+1)} m(\eta)\\
\rho &= \rho_o \psi(\eta)\\
H &= H_o R^n g(\eta)
\end{aligned}\right] \qquad (7.31)$$

INITIAL CONDITION AND SOLUTIONS

Let the explosion take place at a point at t=0 then,

$u = \infty$ for $r \to 0$

$u = 0$ for $r \neq 0$

and also as obtained earlier $R \to 0$ for any admissible solution, p, ρ, H and u are all finite for $t \to 0$ everywhere in $r \leq R$ by putting the value of n=-3/2 in (7.26) we get the shock radius R as

$$R = \beta t^{2/5} \tag{7.32}$$

β is an absolute constant from (7.32) the shock velocity is then given by

$$V = \frac{dR}{dt} = \frac{2}{5}(R/t) \tag{7.33}$$

Again by using (7.32) the similarity transformation can be written as

$$\rho = f_1(\xi) \qquad u = t^{-3/5} f_2(\xi)$$

$$P = t^{-6/5} f_3(\xi) \qquad H = t^{-3/5} f_4(\xi)$$

$$F = t^{-9/5} f_5(\xi) \qquad m = t^{-4/5} f_6(\xi) \tag{7.34}$$

$$\xi = r\, t^{-2/5} \tag{7.35}$$

The energy equation can be written in the form

$$\frac{\partial E}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 (UI_T + F) = 0 \tag{7.36}$$

$$E = \tfrac{1}{2} \rho u^2 + \frac{P}{(K-1)} + \frac{H^2}{2} - \frac{Gm\rho}{r} \tag{7.37}$$

$$I_r = \frac{1}{2} pu^2 + \frac{K\rho}{(k-1)} + Ho - \frac{Gm\rho}{r} \qquad (7.38)$$

putting (7.37) into their similarity form we get

$$E = \frac{1}{2} f(\xi)\, t^{-6/5} f_2^2(\xi) + (1/k-1)\, t^{-6/5}$$

$$F_3(\xi) + \frac{1}{2} t^{-6/5} f_4(\xi) - \frac{T^{-6/5} f_6(\xi)\, f(\xi)}{\xi_j} \qquad (7.39)$$

or

$$E = t^{-6/5} f(\xi)$$

where

$$F(\xi) = \frac{1}{2} f1(\xi)\, f2(\xi) + \frac{1}{k-1} f3(\xi j) + \frac{1}{2} f4(\xi I) - \frac{f6(\xi)\, f(\xi)}{(\xi)} \qquad (7.40)$$

from (7.39) we have

$$\frac{\partial E}{\partial r} = f'(\xi)\, t^{-6/5} \qquad (7.41)$$

$$\frac{\partial E}{\partial t} = -\frac{6}{5} t^{-11/5} f(\xi) - t^{-6/5} f'(\xi) \frac{2}{5} r\, t^{-7/5} \qquad (7.42)$$

with the help of (7.39), (7.41) , (7.42) we get

$$\frac{\partial E}{\partial t} + \frac{6}{5t} E_T + \frac{2r}{5t} \frac{\partial E}{\partial r} = 0 \qquad (7.43)$$

$$\frac{\partial E}{\partial t} + \frac{2}{5tr^2} + \frac{\partial r^3}{\partial r} E = 0 \qquad (7.44)$$

from (7.44) we have

$$\frac{\partial}{\partial r} + t^{12} (UI_T + F) = \frac{\partial}{\partial r} \frac{2r\,E}{5t} \qquad (7.45)$$

$$(UI_T + F) = \frac{2r\,E}{R}$$

where we have taken the constant of integration to be zero and the value of V is substituted from (7.33). The equation (7.45) can also be written as

$$U' = \frac{x(^{1/2} V^2 \rho u^2 + \frac{\rho}{k-1} + \frac{H^2}{2} \frac{Gmp}{R_x}}{t\, 1/2\, v^2 \rho u^{12} + \frac{K\rho}{k-1} + \rho - \frac{Gm\rho}{R_x} - \frac{F}{VI_r}} \qquad (7.46)$$

where $U = vu'$, $\qquad r = R_x$

from (7.46) we have

$$\frac{p}{\rho} = \frac{k-1}{(ku'-x)} [c_1 u^{12} (x-u') + c_2 \rho (x-2u') + \frac{c_3 m}{x}(u'-x) - c_4 f] \qquad (7.47)$$

where c_1, c_2, c_3 and c_4 are function of time and $H = c\rho$ as a consquence of (7.2) and (7.3) from (7.2) and (7.11) we obtain

$$\frac{1}{p}\frac{\partial p}{\partial r} - \frac{(k-1)}{\rho}\frac{\partial \rho}{\partial r} = -\frac{1}{pu}\frac{\partial p}{\partial t} + \frac{(k-1)}{\partial u}\frac{\partial \rho}{\partial t} - \frac{k-1}{r^2}\frac{\partial (fr^2)}{\partial r} - \frac{1}{u}\frac{\partial u}{\partial r} - \frac{2}{r} \qquad (7.48)$$

by putting the value of ξ and v in (7.34) we get the relations

$$\frac{\partial \rho}{\partial t} = \frac{rv}{R}\frac{\partial \rho}{\partial r} \qquad (7.49)$$

$$\frac{\partial p}{\partial t} = \frac{3pv}{R} - \frac{rv}{\partial r}\frac{\partial p}{\partial r} \tag{7.50}$$

by using (7.49) and (7.51) we can write equation (7.48)

$$\frac{1}{p}\frac{\partial p}{\partial r} - \frac{k-1}{\rho}\frac{\partial \rho}{\partial r} = -\frac{2}{r}\frac{\left(\frac{1}{v}\frac{\partial u}{\partial r} - \frac{1}{R}\right)}{\left(\frac{u}{v} - \frac{r}{R}\right)} - \frac{(k-1)u}{v\left(\frac{u}{v} - \frac{r}{R}\right)r^2}\frac{\partial fr^2}{\partial r}$$

(7.51)

integrating this we get

$$\frac{p}{\rho^{(k-1)}} = \frac{c_2}{r^2\left(\frac{u}{v} - \frac{r}{R}\right)} - {}_{\exp}\int \frac{(k-1)u}{v\left(\frac{u}{v} - \frac{r}{R}\right)r^2}\frac{\partial (fx^2)}{\partial r}dr \tag{7.52}$$

substituting (7.47) in (7.53) we have

$$\frac{p}{\rho^{(k-1)}} = \frac{c_2}{x^2(u' - x)} - \exp\int \frac{(k-1)u'}{(u' - x)x^2}\frac{\partial (fx^2)}{\partial x}dx \tag{7.53}$$

Eliminating P between (7.53) and (7.47) and dropping the primes. We obtain the equation

$$\rho^{(3-k)}(\gamma - 2u) + \rho^{(2-k)}\{D_1 u^2 (\gamma - u + \frac{D^2 m}{r}(u - \gamma - D_3 F\}$$

$$x\frac{D_4(ku - \gamma)}{\gamma^2(u - \gamma)} - D_5(k^u - \gamma)\int_{\exp}\frac{(k-1)u}{(u - \gamma)\gamma^2}\frac{d(f\gamma^2)}{d\gamma}d\gamma \tag{7.54}$$

which determines ρ. For simplicity we can write total derivatives in place at partial ones. D_1, D_2, D_3, D_4 and D_5 being constants depending upon time.

Similarly eliminating ρ between (7.53) and (7.47) we can get

$$P = \frac{(k-2)/\rho^{(k-1)} \quad A_1(\gamma-2u)\, p^{(1/k-1)} \quad 2/k-1}{(ku-\gamma) - \left\{\dfrac{A_2}{\gamma^2(u-\gamma)} - \int_{\exp} \dfrac{(k-1)ue}{(u-\gamma)\gamma^2} \quad \dfrac{d(f\gamma^2)}{d\gamma}\, dx \right\}}$$

$$\times \frac{\left(A_3 \quad U^2(\gamma-u) - \quad A_4\left(\quad \dfrac{m}{\gamma}(\gamma-u)\right) - A_5 F\right.}{(ku-\gamma)}$$

$$\times \frac{A_6}{\gamma^2(u-\gamma)} - \int_{\exp} \frac{(k-1)u}{(u-\gamma)\gamma^2} \quad \frac{d(f\gamma^2)}{d\gamma}\, d\gamma \qquad -1/(k-1) \qquad (7.55)$$

which determines P, A_1, A_2, A_3, A_4 and A_5 are constants depending on time. As a consequence of (7.2), (7.4) and (7.54) we write

$$H^{(3-k)}(\gamma-2u) + H^{(2-k)}\left(B_1 u^2(\gamma-u) + \frac{B_2 m}{\gamma}(u-\gamma) - B_3 F\right.$$

$$\times B_4 - \frac{Ku-\gamma}{\gamma^2(u-\gamma)} - B_5(Ku-\gamma)\int_{\exp} \frac{(k-1)u}{(u-\gamma)\gamma^2} - \quad \frac{d(f\gamma^2)}{d\gamma}\, d\gamma \qquad (7.56)$$

where B_1, B_2, B_3, B_4 and B_5 are constants depending on time. From equation (7.2) on using (7.47) and (7.48) we get

$$\frac{1}{\rho}\left(\frac{d\rho}{dr}\right) = \frac{1}{r-u}\left(\frac{du}{dr} + \frac{2u}{r}\right) \qquad (7.57)$$

Differentiating (7.55) with respect to r and then using (7.57) we get

$$\rho^{3-k}\left\{\frac{(3-k)\,(r-2u)\,(du/dr+2u/r)}{r-u} + (1-2du/dr) + \right.$$

$$\rho^{2-k}\left[\frac{(2-k)\,(du/dt+2u/r)}{r-u}\,(D_1\,u^2(r-u) + \frac{D_3 m(u-r)}{r} - D_3F\right\} + \frac{d}{dr}(D_1u^2\,(r-u) +$$

$$\frac{D_2\,m\,(u-r)}{r} - D_2F]$$

$$= \frac{d}{dr}\left[\frac{D_4(ku-r)}{r^2(u-r)} - D_2(ku-r)\,{}_{\exp}\!\int\frac{(k-1)u}{(u-r)r^2}\,\frac{d(fr^2)}{dr}dr\right] \qquad (7.58)$$

The equation (7.58) and (7.54) express relationship between u and r then (7.54) (7.55) & (7.56) express ρ, p and H in terms of r and therefore, given the required solution.

In the absence of any radiation effects. The klimshin's cofficient K becomes the usual adiabatic exponent r and then the solution referred above agrees with corresponding solution in this chapter.

REFERENCES

1. Carrus P. A., Fox P. A., Haas F. & Kopal Z Astrophysics J. 113,193 (1951)

2. L I Sedov : Similarity and dimensional method in mechanics
Academic Press, New York (1959)

3. B. G. Verma Zamp Vol. 20, 119 (1969)

4. Johan H Lee : Gas Dynamics of Detonation
AFOSAR Grant 69, 1752 (1978)

5. R. L. Rabie & G. R. Fowles : The poly Morphic Detonation
Phy. fluids 22(3) 422 - 435 (1979)